IQI 04, Seminar 6

Produced with pdflatex and xfig

- Distinguishable states of qubits.
- Overlaps.
- Bra-ket algebra.
- Measuring one of n qubits.
- Projective measurement.
- Rotation by state preparation.
- Eigenspace measurement.

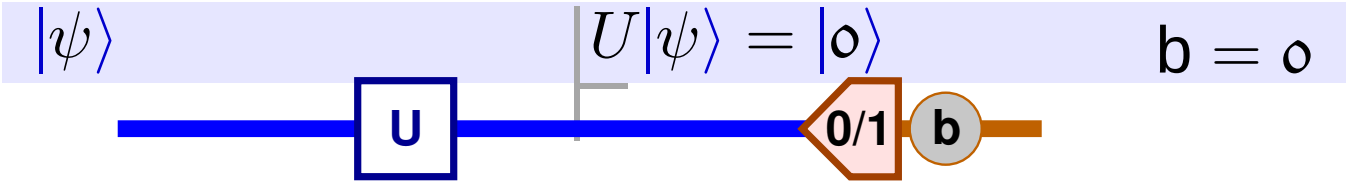
E. “Manny” Knill: knill@boulder.nist.gov

Distinguishable One-Qubit States

- $|\psi\rangle$ and $|\phi\rangle$ are distinguishable if for some unitary U, $U|\psi\rangle = |\mathtt{0}\rangle$ and $U|\phi\rangle = |\mathtt{1}\rangle$.

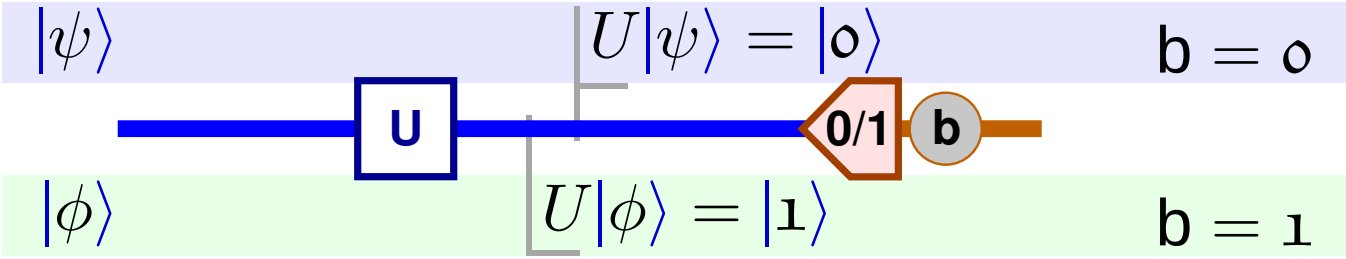
Distinguishable One-Qubit States

- $|\psi\rangle$ and $|\phi\rangle$ are distinguishable if for some unitary U, $U|\psi\rangle = |0\rangle$ and $U|\phi\rangle = |1\rangle$.



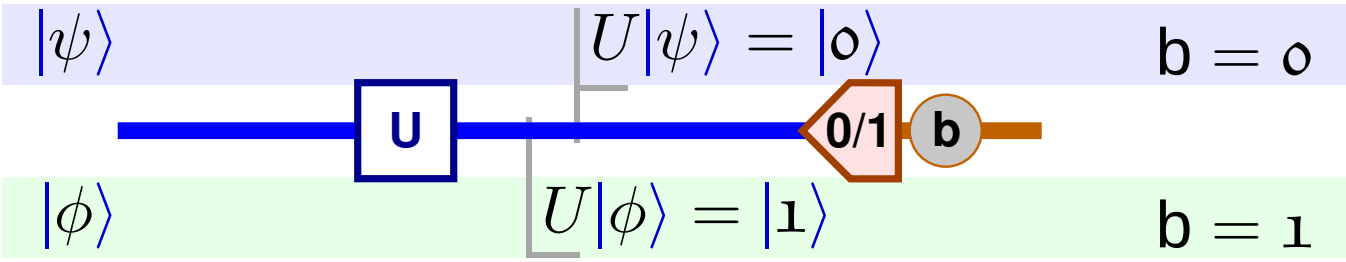
Distinguishable One-Qubit States

- $|\psi\rangle$ and $|\phi\rangle$ are distinguishable if for some unitary U, $U|\psi\rangle = |0\rangle$ and $U|\phi\rangle = |1\rangle$.



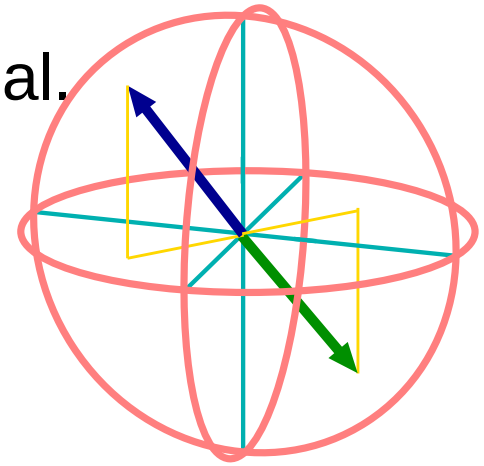
Distinguishable One-Qubit States

- $|\psi\rangle$ and $|\phi\rangle$ are distinguishable if for some unitary U, $U|\psi\rangle = |0\rangle$ and $U|\phi\rangle = |1\rangle$.



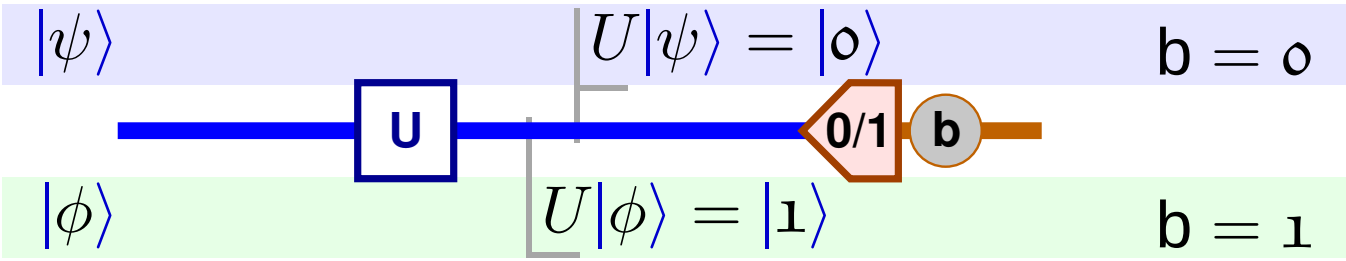
- Distinguishable states of one qubit are antipodal.

. . . by universality of rotations.



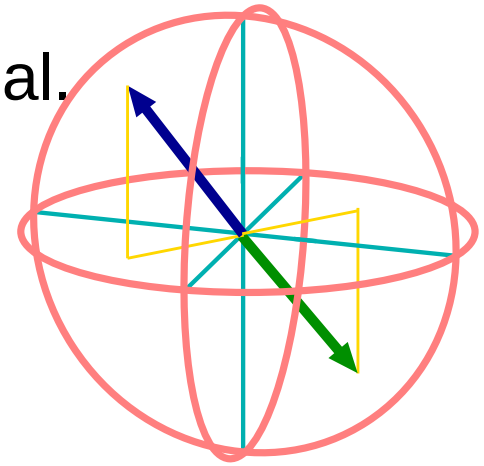
Distinguishable One-Qubit States

- $|\psi\rangle$ and $|\phi\rangle$ are distinguishable if for some unitary U, $U|\psi\rangle = |0\rangle$ and $U|\phi\rangle = |1\rangle$.



- Distinguishable states of one qubit are antipodal.

  . . . by universality of rotations.

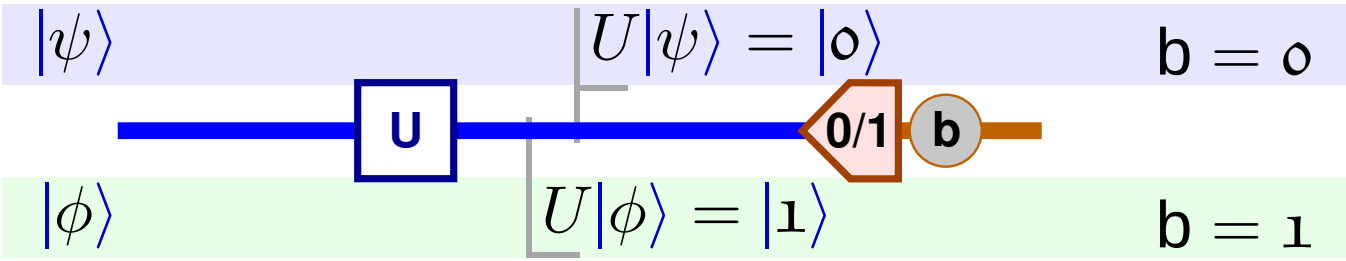


- Distinguishable states are orthogonal.

  . . . by unitarity of U.

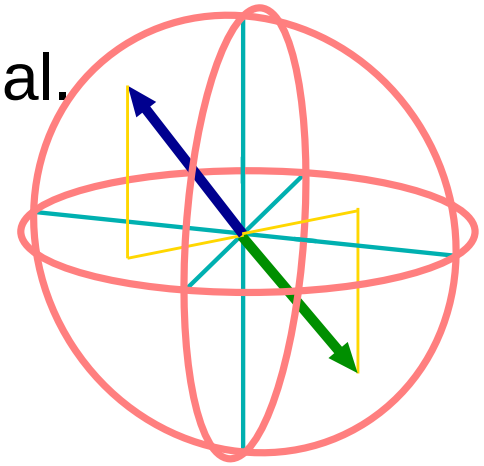
Distinguishable One-Qubit States

- $|\psi\rangle$ and $|\phi\rangle$ are distinguishable if for some unitary U, $U|\psi\rangle = |\mathtt{0}\rangle$ and $U|\phi\rangle = |\mathtt{1}\rangle$.



- Distinguishable states of one qubit are antipodal.

  . . . by universality of rotations.



- Distinguishable states are orthogonal.

  . . . by unitarity of U.

Write $|\psi\rangle = \alpha|\mathtt{0}\rangle + \beta|\mathtt{1}\rangle$ and $|\phi\rangle = \gamma|\mathtt{0}\rangle + \delta|\mathtt{1}\rangle$.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (\bar{\alpha}, \bar{\beta}) \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = 0$$

Bra-Ket Algebra

- Need to be able to “dagger” kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger |\phi\rangle \equiv {}^{(\bar{\alpha},\bar{\beta})} \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = 0$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger |\phi\rangle \equiv {}^{(\bar\alpha,\,\bar\beta)} \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\langle\psi|\phi\rangle = \langle\psi||\phi\rangle \quad = \quad (|\psi\rangle)^\dagger|\phi\rangle$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi||\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= (\alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle)^\dagger|\phi\rangle\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi||\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= (\alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle)^\dagger|\phi\rangle \\ &= (\bar\alpha\langle\mathfrak{o}| + \bar\beta\langle\mathfrak{1}|)|\phi\rangle\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi||\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= (\bar\alpha\langle\mathfrak{o}| + \bar\beta\langle\mathfrak{1}|)|\phi\rangle\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi||\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= (\bar\alpha\langle\mathfrak{o}| + \bar\beta\langle\mathfrak{1}|)|\phi\rangle\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi||\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle\\ &= (\bar\alpha\langle\mathfrak{o}| + \bar\beta\langle\mathfrak{1}|)|\phi\rangle\\ &= (\bar\alpha\langle\mathfrak{o}| + \bar\beta\langle\mathfrak{1}|)(\gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle)\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi||\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= (\bar\alpha\langle\mathfrak{o}| + \bar\beta\langle\mathfrak{1}|)(\gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle)\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi||\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= (\bar\alpha\langle\mathfrak{o}| + \bar\beta\langle\mathfrak{1}|)(\gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle)\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv (\bar{\alpha}, \bar{\beta}) \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar{\alpha}\langle\psi| + \bar{\beta}\langle\phi|$.
  - $\langle\mathfrak{a}\|\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned} \langle\psi|\phi\rangle = \langle\psi\|\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= (\bar{\alpha}\langle\mathfrak{o}| + \bar{\beta}\langle\mathfrak{1}|)(\gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle) \\ &= \bar{\alpha}\gamma\langle\mathfrak{o}\|\mathfrak{o}\rangle + \bar{\beta}\gamma\langle\mathfrak{1}\|\mathfrak{o}\rangle + \bar{\alpha}\delta\langle\mathfrak{o}\|\mathfrak{1}\rangle + \bar{\beta}\delta\langle\mathfrak{1}\|\mathfrak{1}\rangle \end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv (\bar{\alpha}, \bar{\beta}) \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar{\alpha}\langle\psi| + \bar{\beta}\langle\phi|$.
  - $\langle\mathfrak{a}\|\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned} \langle\psi|\phi\rangle = \langle\psi\|\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= \bar{\alpha}\gamma\langle\mathfrak{o}\|\mathfrak{o}\rangle + \bar{\beta}\gamma\langle\mathfrak{1}\|\mathfrak{o}\rangle + \bar{\alpha}\delta\langle\mathfrak{o}\|\mathfrak{1}\rangle + \bar{\beta}\delta\langle\mathfrak{1}\|\mathfrak{1}\rangle \end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}\|\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi\|\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= \bar\alpha\gamma\langle\mathfrak{o}\|\mathfrak{o}\rangle + \bar\beta\gamma\langle\mathfrak{1}\|\mathfrak{o}\rangle + \bar\alpha\delta\langle\mathfrak{o}\|\mathfrak{1}\rangle + \bar\beta\delta\langle\mathfrak{1}\|\mathfrak{1}\rangle\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}\|\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi\|\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= \bar\alpha\gamma\underbrace{\langle\mathfrak{o}\|\mathfrak{o}\rangle}_{1} + \bar\beta\gamma\langle\mathfrak{1}\|\mathfrak{o}\rangle + \bar\alpha\delta\langle\mathfrak{o}\|\mathfrak{1}\rangle + \bar\beta\delta\langle\mathfrak{1}\|\mathfrak{1}\rangle\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}\|\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi\|\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= \bar\alpha\gamma\underbrace{\langle\mathfrak{o}\|\mathfrak{o}\rangle}_{1} + \bar\beta\gamma\underbrace{\langle\mathfrak{1}\|\mathfrak{o}\rangle}_{0} + \bar\alpha\delta\langle\mathfrak{o}\|\mathfrak{1}\rangle + \bar\beta\delta\langle\mathfrak{1}\|\mathfrak{1}\rangle\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}\|\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi\|\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= \bar\alpha\gamma\underbrace{\langle\mathfrak{o}\|\mathfrak{o}\rangle}_{1} + \bar\beta\gamma\underbrace{\langle\mathfrak{1}\|\mathfrak{o}\rangle}_{0} + \bar\alpha\delta\underbrace{\langle\mathfrak{o}\|\mathfrak{1}\rangle}_{0} + \bar\beta\delta\langle\mathfrak{1}\|\mathfrak{1}\rangle\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}\|\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi\|\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= \bar\alpha\gamma\underbrace{\langle\mathfrak{o}\|\mathfrak{o}\rangle}_{1} + \bar\beta\gamma\underbrace{\langle\mathfrak{1}\|\mathfrak{o}\rangle}_{0} + \bar\alpha\delta\underbrace{\langle\mathfrak{o}\|\mathfrak{1}\rangle}_{0} + \bar\beta\delta\underbrace{\langle\mathfrak{1}\|\mathfrak{1}\rangle}_{1}\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger|\phi\rangle \equiv {}^{(\bar\alpha,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}\|\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Example: $|\psi\rangle = \alpha|\mathfrak{o}\rangle + \beta|\mathfrak{1}\rangle, |\phi\rangle = \gamma|\mathfrak{o}\rangle + \delta|\mathfrak{1}\rangle$

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi\|\phi\rangle &= (|\psi\rangle)^\dagger|\phi\rangle \\ &= \bar\alpha\gamma \qquad\qquad\qquad\qquad + \bar\beta\delta\end{aligned}$$

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger |\phi\rangle \equiv {}^{(\bar\alpha,\,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.

Bra-Ket Algebra

- Need to be able to "dagger" kets.

$$|\psi\rangle \perp |\phi\rangle \quad \text{iff} \quad (|\psi\rangle)^\dagger |\phi\rangle \equiv {}^{(\bar\alpha,\,\bar\beta)}\begin{pmatrix}\gamma\\ \delta\end{pmatrix} = 0$$

Write $\langle\psi| \doteq (|\psi\rangle)^\dagger$. Meaning derived from:

- Bra-ket rules.
  - $|\mathfrak{o}\rangle^\dagger = \langle\mathfrak{o}|, |\mathfrak{1}\rangle^\dagger = \langle\mathfrak{1}|$.
  - $\big(\alpha|\psi\rangle + \beta|\phi\rangle\big)^\dagger = \bar\alpha\langle\psi| + \bar\beta\langle\phi|$.
  - $\langle\mathfrak{a}||\mathfrak{b}\rangle \doteq \langle\mathfrak{a}|\mathfrak{b}\rangle = \delta_{\mathfrak{b}\mathfrak{a}}$.
  - Product distributes over addition.
  - For labeled systems: $|\psi\rangle_{\mathsf{A}}^\dagger = {}^{\mathsf{A}}\langle\psi|$.
  - Labeled expressions involving disjoint systems commute.

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle \;=\; |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$\begin{aligned}\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\ &= |0\rangle\langle 0|0\rangle - |1\rangle\langle 1|0\rangle\end{aligned}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$\begin{aligned}\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\ &= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\langle 1|0\rangle\end{aligned}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$\begin{aligned}\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\ &= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0}\end{aligned}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$\begin{aligned}
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\
&= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0} \\
&= |0\rangle
\end{aligned}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$
\begin{aligned}
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\
&= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0} \\
&= |0\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|1\rangle &= |0\rangle\langle 0||1\rangle - |1\rangle\langle 1||1\rangle
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$
\begin{aligned}
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\
&= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0} \\
&= |0\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|1\rangle &= |0\rangle\langle 0||1\rangle - |1\rangle\langle 1||1\rangle \\
&= |0\rangle\langle 0|1\rangle - |1\rangle\langle 1|1\rangle
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$
\begin{aligned}
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\
&= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0} \\
&= |0\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|1\rangle &= |0\rangle\langle 0||1\rangle - |1\rangle\langle 1||1\rangle \\
&= |0\rangle\underbrace{\langle 0|1\rangle}_{0} - |1\rangle\langle 1|1\rangle
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$
\begin{aligned}
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\
&= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0} \\
&= |0\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|1\rangle &= |0\rangle\langle 0||1\rangle - |1\rangle\langle 1||1\rangle \\
&= |0\rangle\underbrace{\langle 0|1\rangle}_{0} - |1\rangle\underbrace{\langle 1|1\rangle}_{1}
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$
\begin{aligned}
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\
&= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0} \\
&= |0\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|1\rangle &= |0\rangle\langle 0||1\rangle - |1\rangle\langle 1||1\rangle \\
&= |0\rangle\underbrace{\langle 0|1\rangle}_{0} - |1\rangle\underbrace{\langle 1|1\rangle}_{1} \\
&= -|1\rangle
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$
\begin{aligned}
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\
&= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0} \\
&= |0\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|1\rangle &= |0\rangle\langle 0||1\rangle - |1\rangle\langle 1||1\rangle \\
&= |0\rangle\underbrace{\langle 0|1\rangle}_{0} - |1\rangle\underbrace{\langle 1|1\rangle}_{1} \\
&= -|1\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)(\alpha|0\rangle + \beta|1\rangle) &= \alpha|0\rangle - \beta|1\rangle
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.

  Example: Apply $|0\rangle\langle 0| - |1\rangle\langle 1|$ to states.

$$
\begin{aligned}
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|0\rangle &= |0\rangle\langle 0||0\rangle - |1\rangle\langle 1||0\rangle \\
&= |0\rangle\underbrace{\langle 0|0\rangle}_{1} - |1\rangle\underbrace{\langle 1|0\rangle}_{0} \\
&= |0\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)|1\rangle &= |0\rangle\langle 0||1\rangle - |1\rangle\langle 1||1\rangle \\
&= |0\rangle\underbrace{\langle 0|1\rangle}_{0} - |1\rangle\underbrace{\langle 1|1\rangle}_{1} \\
&= -|1\rangle \\
\big(|0\rangle\langle 0| - |1\rangle\langle 1|\big)(\alpha|0\rangle + \beta|1\rangle) &= \alpha|0\rangle - \beta|1\rangle
\end{aligned}
$$

  Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) \quad = \quad \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big)$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$\begin{aligned}\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) &= \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big) \\ &= \alpha|1\rangle\langle 0||0\rangle + \beta|1\rangle\langle 0||1\rangle + \alpha|0\rangle\langle 1||0\rangle + \beta|0\rangle\langle 1||1\rangle\end{aligned}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$\begin{aligned}
\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) &= \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big) \\
&= \alpha|1\rangle\langle 0||0\rangle + \beta|1\rangle\langle 0||1\rangle + \alpha|0\rangle\langle 1||0\rangle + \beta|0\rangle\langle 1||1\rangle \\
&= \alpha|1\rangle\langle 0|0\rangle + \beta|1\rangle\langle 0|1\rangle + \alpha|0\rangle\langle 1|0\rangle + \beta|0\rangle\langle 1|1\rangle
\end{aligned}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$\begin{aligned}
\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) &= \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big) \\
&= \alpha|1\rangle\langle 0||0\rangle + \beta|1\rangle\langle 0||1\rangle + \alpha|0\rangle\langle 1||0\rangle + \beta|0\rangle\langle 1||1\rangle \\
&= \alpha|1\rangle\underbrace{\langle 0|0\rangle}_{1} + \beta|1\rangle\langle 0|1\rangle + \alpha|0\rangle\langle 1|0\rangle + \beta|0\rangle\langle 1|1\rangle
\end{aligned}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$
\begin{aligned}
\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) &= \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big) \\
&= \alpha|1\rangle\langle 0||0\rangle + \beta|1\rangle\langle 0||1\rangle + \alpha|0\rangle\langle 1||0\rangle + \beta|0\rangle\langle 1||1\rangle \\
&= \alpha|1\rangle\underbrace{\langle 0|0\rangle}_{1} + \beta|1\rangle\underbrace{\langle 0|1\rangle}_{0} + \alpha|0\rangle\langle 1|0\rangle + \beta|0\rangle\langle 1|1\rangle
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$
\begin{aligned}
\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) &= \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big) \\
&= \alpha|1\rangle\langle 0||0\rangle + \beta|1\rangle\langle 0||1\rangle + \alpha|0\rangle\langle 1||0\rangle + \beta|0\rangle\langle 1||1\rangle \\
&= \alpha|1\rangle\underbrace{\langle 0|0\rangle}_{1} + \beta|1\rangle\underbrace{\langle 0|1\rangle}_{0} + \alpha|0\rangle\underbrace{\langle 1|0\rangle}_{0} + \beta|0\rangle\langle 1|1\rangle
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$
\begin{aligned}
\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) &= \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big) \\
&= \alpha|1\rangle\langle 0||0\rangle + \beta|1\rangle\langle 0||1\rangle + \alpha|0\rangle\langle 1||0\rangle + \beta|0\rangle\langle 1||1\rangle \\
&= \alpha|1\rangle\underbrace{\langle 0|0\rangle}_{1} + \beta|1\rangle\underbrace{\langle 0|1\rangle}_{0} + \alpha|0\rangle\underbrace{\langle 1|0\rangle}_{0} + \beta|0\rangle\underbrace{\langle 1|1\rangle}_{1}
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$
\begin{aligned}
\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) &= \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big) \\
&= \alpha|1\rangle\langle 0||0\rangle + \beta|1\rangle\langle 0||1\rangle + \alpha|0\rangle\langle 1||0\rangle + \beta|0\rangle\langle 1||1\rangle \\
&= \alpha|1\rangle\underbrace{\langle 0|0\rangle}_{1} + \beta|1\rangle\underbrace{\langle 0|1\rangle}_{0} + \alpha|0\rangle\underbrace{\langle 1|0\rangle}_{0} + \beta|0\rangle\underbrace{\langle 1|1\rangle}_{1} \\
&= \alpha|1\rangle + \beta|0\rangle
\end{aligned}
$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$

$$\begin{aligned}
\sigma_x\big(\alpha|0\rangle + \beta|1\rangle\big) &= \big(|1\rangle\langle 0| + |0\rangle\langle 1|\big)\big(\alpha|0\rangle + \beta|1\rangle\big) \\
&= \alpha|1\rangle\langle 0||0\rangle + \beta|1\rangle\langle 0||1\rangle + \alpha|0\rangle\langle 1||0\rangle + \beta|0\rangle\langle 1||1\rangle \\
&= \alpha|1\rangle\underbrace{\langle 0|0\rangle}_{1} + \beta|1\rangle\underbrace{\langle 0|1\rangle}_{0} + \alpha|0\rangle\underbrace{\langle 1|0\rangle}_{0} + \beta|0\rangle\underbrace{\langle 1|1\rangle}_{1} \\
&= \alpha|1\rangle + \beta|0\rangle
\end{aligned}$$

Ket-Bra Expressions

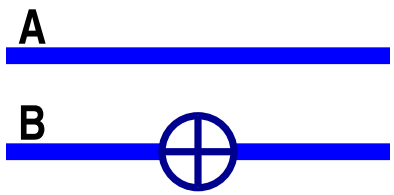
- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathrm{B})} = |0\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 1| + |1\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$

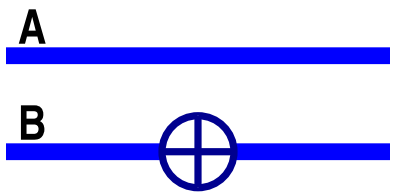


Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

Ex.: $\sigma_x{}^{(\mathsf{B})}\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



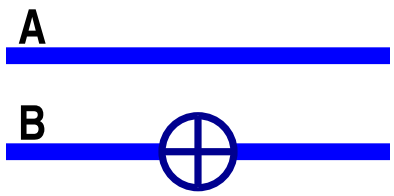
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

Ex.: $\sigma_x{}^{(\mathsf{B})}\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$

$= \left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right)\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$

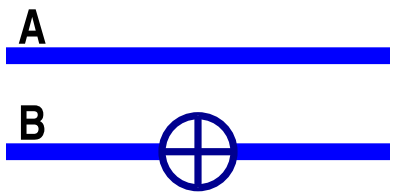
$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



Ex.: $\sigma_x{}^{(\mathsf{B})}\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$

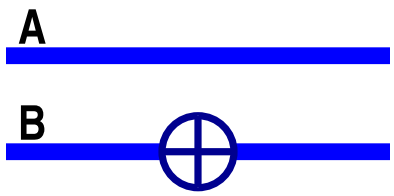
$$= \left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right)\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||00\rangle_{\mathsf{AB}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||00\rangle_{\mathsf{AB}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||11\rangle_{\mathsf{AB}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||11\rangle_{\mathsf{AB}}\right)$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



Ex.: $\sigma_x{}^{(\mathsf{B})}\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$

$$= \left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right)\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$$

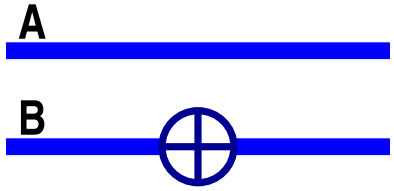
$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||00\rangle_{\mathsf{AB}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||00\rangle_{\mathsf{AB}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||11\rangle_{\mathsf{AB}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||11\rangle_{\mathsf{AB}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}\right)$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



Ex.: $\sigma_x{}^{(\mathsf{B})}\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$

$$= \left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right)\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$$

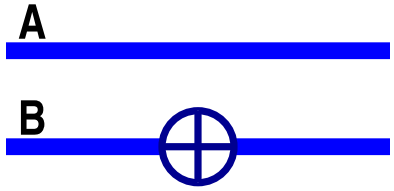
$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||00\rangle_{\mathsf{AB}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||00\rangle_{\mathsf{AB}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||11\rangle_{\mathsf{AB}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||11\rangle_{\mathsf{AB}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}\right)$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



Ex.: $\sigma_x{}^{(\mathsf{B})}\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$

$$= \left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right)\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||00\rangle_{\mathsf{AB}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||00\rangle_{\mathsf{AB}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||11\rangle_{\mathsf{AB}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||11\rangle_{\mathsf{AB}}\right)$$

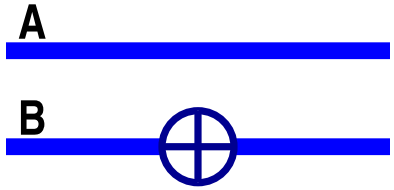
$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}{}^{\mathsf{B}}\langle 1|0\rangle_{\mathsf{B}} + |0\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}{}^{\mathsf{B}}\langle 0|0\rangle_{\mathsf{B}} + i|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}{}^{\mathsf{B}}\langle 1|1\rangle_{\mathsf{B}} + i|1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}{}^{\mathsf{B}}\langle 0|1\rangle_{\mathsf{B}}\right.$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



Ex.: $\sigma_x^{(\mathsf{B})}\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$

$$= \left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right)\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{\mathsf{AB}} + i|11\rangle_{\mathsf{AB}}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||00\rangle_{\mathsf{AB}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||00\rangle_{\mathsf{AB}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||11\rangle_{\mathsf{AB}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||11\rangle_{\mathsf{AB}}\right)$$

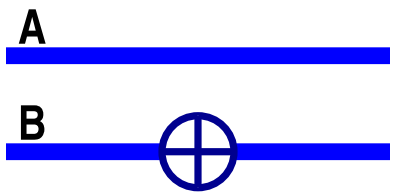
$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + i|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1||1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}} + i|1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0||1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}\underbrace{{}^{\mathsf{B}}\langle 1|0\rangle_{\mathsf{B}}}_{0} + |0\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}\underbrace{{}^{\mathsf{B}}\langle 0|0\rangle_{\mathsf{B}}}_{1} + i|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}\underbrace{{}^{\mathsf{B}}\langle 1|1\rangle_{\mathsf{B}}}_{1} + i|1\rangle_{\mathsf{A}}|1\rangle_{\mathsf{B}}\underbrace{{}^{\mathsf{B}}\langle 0|1\rangle_{\mathsf{B}}}_{0}\right)$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x^{(B)} = |0\rangle_B^B\langle 1| + |1\rangle_B^B\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



Ex.: $\sigma_x^{(B)}\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + i|11\rangle_{AB}\right)\right)$

$$= \left(|0\rangle_B^B\langle 1| + |1\rangle_B^B\langle 0|\right)\left(\frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + i|11\rangle_{AB}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_B^B\langle 1||00\rangle_{AB} + |1\rangle_B^B\langle 0||00\rangle_{AB} + i|0\rangle_B^B\langle 1||11\rangle_{AB} + i|1\rangle_B^B\langle 0||11\rangle_{AB}\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle_B^B\langle 1||0\rangle_A|0\rangle_B + |1\rangle_B^B\langle 0||0\rangle_A|0\rangle_B + i|0\rangle_B^B\langle 1||1\rangle_A|1\rangle_B + i|1\rangle_B^B\langle 0||1\rangle_A|1\rangle_B\right)$$

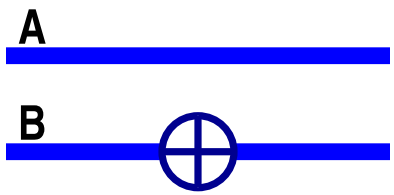
$$= \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B\underbrace{{}^B\langle 1|0\rangle_B}_{0} + |0\rangle_A|1\rangle_B\underbrace{{}^B\langle 0|0\rangle_B}_{1} + i|1\rangle_A|0\rangle_B\underbrace{{}^B\langle 1|1\rangle_B}_{1} + i|1\rangle_A|1\rangle_B\underbrace{{}^B\langle 0|1\rangle_B}_{0}\right)$$

$$= \frac{1}{\sqrt{2}}\left(|01\rangle_{AB} + i|10\rangle_{AB}\right)$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathrm{B})} = |0\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 1| + |1\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$

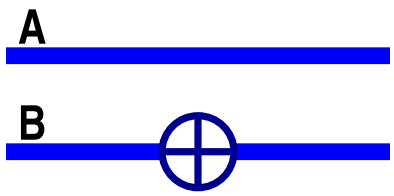


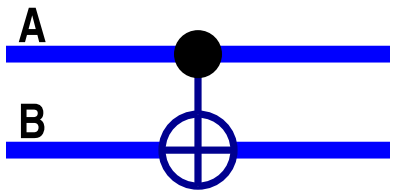
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.
  - $\mathbf{cnot}$ from A to B:

$$\begin{aligned}\mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right)\end{aligned}$$

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$





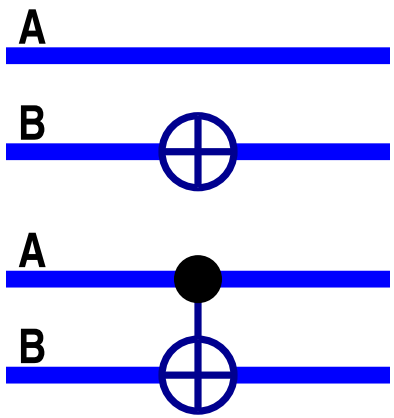
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.
  - $\mathbf{cnot}$ from A to B:

$$\begin{aligned}\mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\big(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\big)\end{aligned}$$

Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\big(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\big)|0\rangle_{\mathsf{B}}$

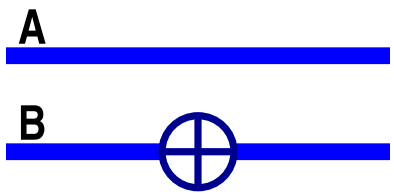
$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



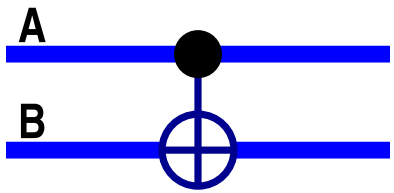
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right) \end{aligned}$$

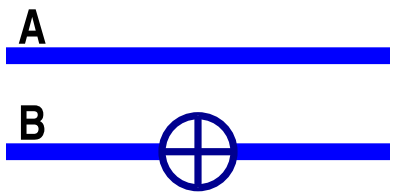
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\left(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\right)|0\rangle_{\mathsf{B}}$

$$= \left(|0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}\right)\left(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\right)|0\rangle_{\mathsf{B}}$$

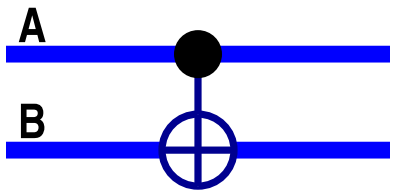
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right) \end{aligned}$$

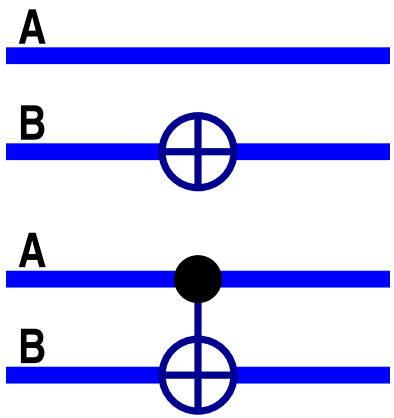
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\left(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\right)|0\rangle_{\mathsf{B}}$

$$\begin{aligned} &= \left(|0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}\right)\left(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\right)|0\rangle_{\mathsf{B}} \\ &= \left(|0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}\right)\left(\alpha|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}\right) \end{aligned}$$

Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.
  - $\mathbf{cnot}$ from A to B:

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right) \end{aligned}$$

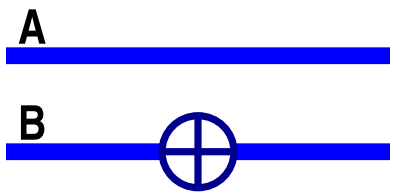
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\left(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\right)|0\rangle_{\mathsf{B}}$

$$= \left(|0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}\right)\left(\alpha|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}\right)$$

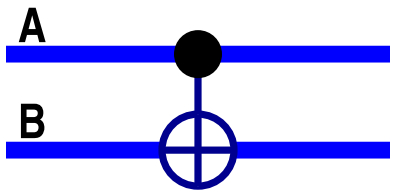
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\big(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\big) \end{aligned}$$

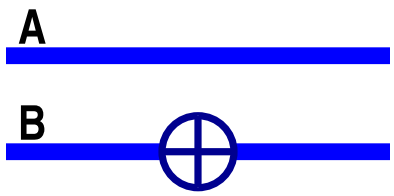
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\big(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\big)|0\rangle_{\mathsf{B}}$

$$= \big(|0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}\big)\big(\alpha|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}\big)$$

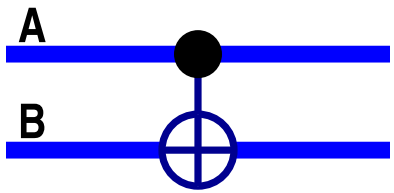
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned}\mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\big(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\big)\end{aligned}$$

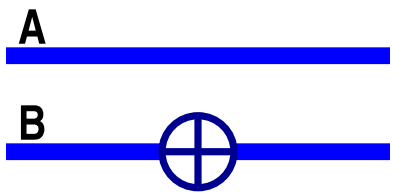
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\big(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\big)|0\rangle_{\mathsf{B}}$

$$\begin{aligned} &= \big(|0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}\big)\big(\alpha|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}\big) \\ &= \alpha|0\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 0||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|0\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 0||1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} \\ &\quad + \alpha|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}\end{aligned}$$

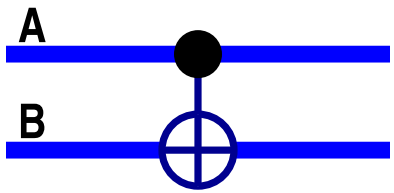
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\big(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\big) \end{aligned}$$

Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\big(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\big)|0\rangle_{\mathsf{B}}$

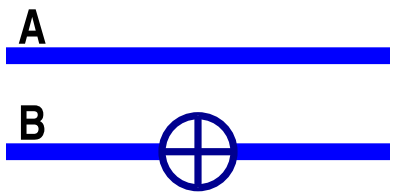
$$= \alpha|0\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 0||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|0\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 0||1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}$$

$$+ \alpha|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}$$

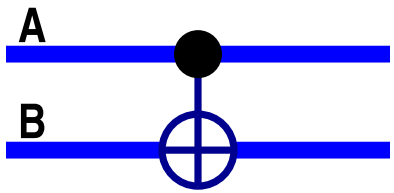
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\big(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\big) \end{aligned}$$

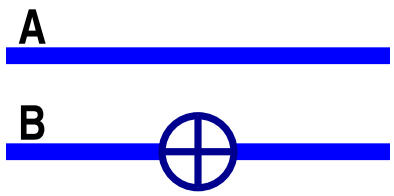
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\big(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\big)|0\rangle_{\mathsf{B}}$

$$\begin{aligned} &= \alpha|0\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 0||0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|0\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 0||1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} \\ &\quad + \alpha|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} \end{aligned}$$

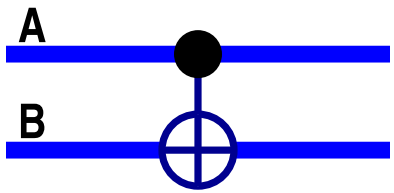
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned}\mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\big(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\big)\end{aligned}$$

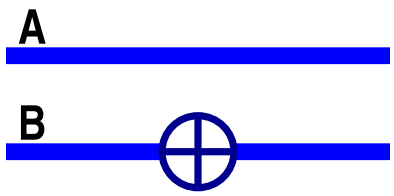
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\big(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\big)|0\rangle_{\mathsf{B}}$

$$\begin{aligned}&= \alpha|0\rangle_{\mathsf{A}}\underbrace{{}^{\mathsf{A}}\langle 0||0\rangle_{\mathsf{A}}}_{1}|0\rangle_{\mathsf{B}} + \beta|0\rangle_{\mathsf{A}}\underbrace{{}^{\mathsf{A}}\langle 0||1\rangle_{\mathsf{A}}}_{0}|0\rangle_{\mathsf{B}} \\ &\quad + \alpha|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}}\end{aligned}$$

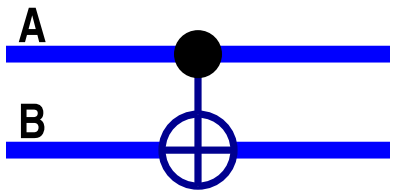
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathrm{B})} = |0\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 1| + |1\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathrm{AB})} &= |0\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 0| + |1\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 1|\sigma_x{}^{(\mathrm{B})} \\ &= |0\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 0| + |1\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 1|\left(|0\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 1| + |1\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 0|\right) \end{aligned}$$

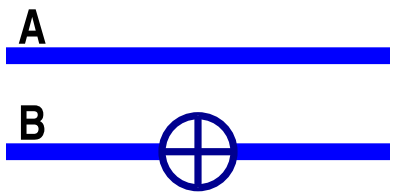
Ex.: $\mathbf{cnot}^{(\mathrm{AB})}\left(\alpha|0\rangle_{\mathrm{A}} + \beta|1\rangle_{\mathrm{A}}\right)|0\rangle_{\mathrm{B}}$

$$\begin{aligned} &= \alpha|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} \\ &\quad + \alpha|1\rangle_{\mathrm{A}}{}^{\mathrm{A}}\langle 1|\,\sigma_x{}^{(\mathrm{B})}|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} + \beta|1\rangle_{\mathrm{A}}{}^{\mathrm{A}}\langle 1|\,\sigma_x{}^{(\mathrm{B})}|1\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} \end{aligned}$$

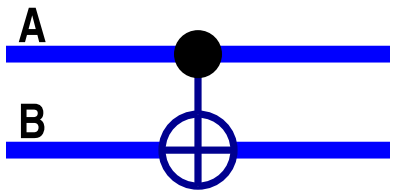
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\big(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\big) \end{aligned}$$

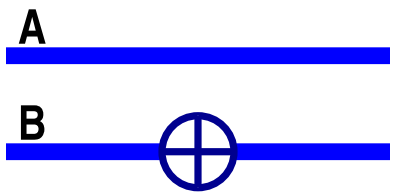
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\big(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\big)|0\rangle_{\mathsf{B}}$

$$\begin{aligned} &= \alpha|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} \\ &\quad + \alpha|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}{}^{\mathsf{A}}\langle 1|\,\sigma_x{}^{(\mathsf{B})}|1\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} \end{aligned}$$

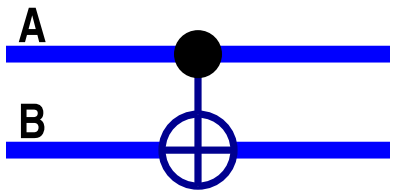
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathrm{B})} = |0\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 1| + |1\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathrm{AB})} &= |0\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 0| + |1\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 1|\sigma_x{}^{(\mathrm{B})} \\ &= |0\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 0| + |1\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 1|\left(|0\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 1| + |1\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 0|\right) \end{aligned}$$

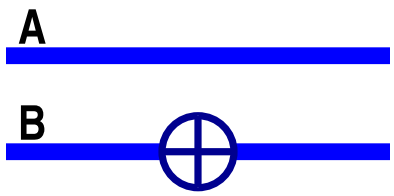
Ex.: $\mathbf{cnot}^{(\mathrm{AB})}\left(\alpha|0\rangle_{\mathrm{A}} + \beta|1\rangle_{\mathrm{A}}\right)|0\rangle_{\mathrm{B}}$

$$\begin{aligned} &= \alpha|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} \\ &\quad + \alpha|1\rangle_{\mathrm{A}}{}^{\mathrm{A}}\langle 1||0\rangle_{\mathrm{A}}\,\sigma_x{}^{(\mathrm{B})}|0\rangle_{\mathrm{B}} + \beta|1\rangle_{\mathrm{A}}{}^{\mathrm{A}}\langle 1||1\rangle_{\mathrm{A}}\,\sigma_x{}^{(\mathrm{B})}|0\rangle_{\mathrm{B}} \end{aligned}$$

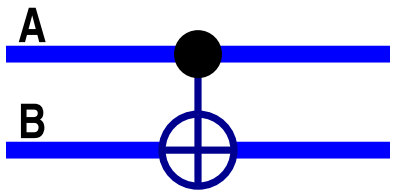
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\big(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\big) \end{aligned}$$

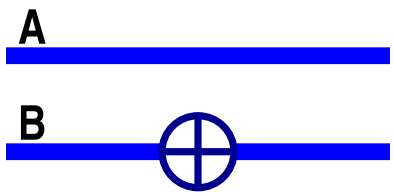
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\big(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\big)|0\rangle_{\mathsf{B}}$

$$\begin{aligned} &= \alpha|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} \\ &\quad + \alpha|1\rangle_{\mathsf{A}}\underbrace{{}^{\mathsf{A}}\langle 1||0\rangle_{\mathsf{A}}}_{0}\,\sigma_x{}^{(\mathsf{B})}|0\rangle_{\mathsf{B}} + \beta|1\rangle_{\mathsf{A}}\underbrace{{}^{\mathsf{A}}\langle 1||1\rangle_{\mathsf{A}}}_{1}\,\sigma_x{}^{(\mathsf{B})}|0\rangle_{\mathsf{B}} \end{aligned}$$

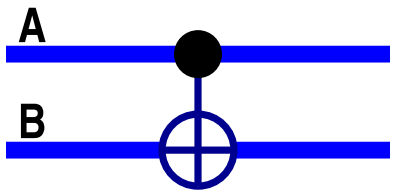
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathsf{B})} = |0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathsf{AB})} &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\sigma_x{}^{(\mathsf{B})} \\ &= |0\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 0| + |1\rangle_{\mathsf{A}}^{\mathsf{A}}\langle 1|\left(|0\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 1| + |1\rangle_{\mathsf{B}}^{\mathsf{B}}\langle 0|\right) \end{aligned}$$

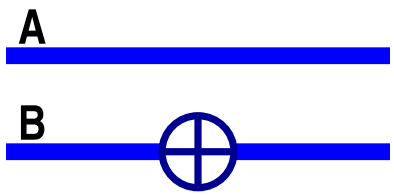
Ex.: $\mathbf{cnot}^{(\mathsf{AB})}\left(\alpha|0\rangle_{\mathsf{A}} + \beta|1\rangle_{\mathsf{A}}\right)|0\rangle_{\mathsf{B}}$

$$\begin{aligned} &= \alpha|0\rangle_{\mathsf{A}}|0\rangle_{\mathsf{B}} \\ &+ \qquad \beta|1\rangle_{\mathsf{A}}\,\sigma_x{}^{(\mathsf{B})}|0\rangle_{\mathsf{B}} \end{aligned}$$

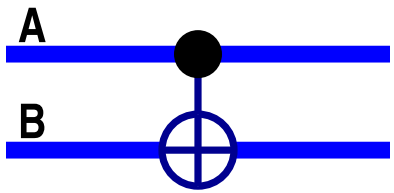
Ket-Bra Expressions

- Ket-Bra expressions are operators.
  - Multiplication by $|0\rangle\langle 0| - |1\rangle\langle 1|$ acts as σ_z.
  - Multiplication by $|0\rangle\langle 1| + |1\rangle\langle 0|$ acts as σ_x.
  - To flip qubit B in system AB multiply by $\sigma_x{}^{(\mathrm{B})} = |0\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 1| + |1\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 0|$.

$$\begin{pmatrix} |0\rangle\langle 0| & |0\rangle\langle 1| \\ |1\rangle\langle 0| & |1\rangle\langle 1| \end{pmatrix}$$



  - $\mathbf{cnot}$ from A to B:



$$\begin{aligned} \mathbf{cnot}^{(\mathrm{AB})} &= |0\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 0| + |1\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 1|\sigma_x{}^{(\mathrm{B})} \\ &= |0\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 0| + |1\rangle_{\mathrm{A}}^{\mathrm{A}}\langle 1|\left(|0\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 1| + |1\rangle_{\mathrm{B}}^{\mathrm{B}}\langle 0|\right) \end{aligned}$$

Ex.: $\mathbf{cnot}^{(\mathrm{AB})}\left(\alpha|0\rangle_{\mathrm{A}} + \beta|1\rangle_{\mathrm{A}}\right)|0\rangle_{\mathrm{B}}$

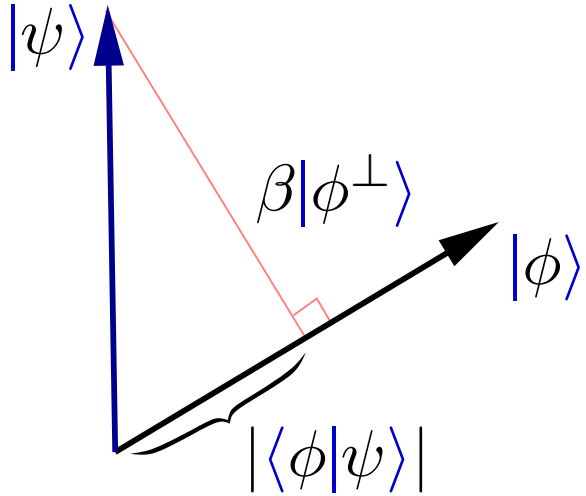
$$\begin{aligned} &= \alpha|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} \\ &\quad + \qquad \beta|1\rangle_{\mathrm{A}}\,\sigma_x{}^{(\mathrm{B})}|0\rangle_{\mathrm{B}} \\ &= \alpha|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} + \beta|1\rangle_{\mathrm{A}}|1\rangle_{\mathrm{B}} \end{aligned}$$

Overlap

- The overlap between $|\psi\rangle$ and $|\phi\rangle$ is $\langle\phi|\psi\rangle$.

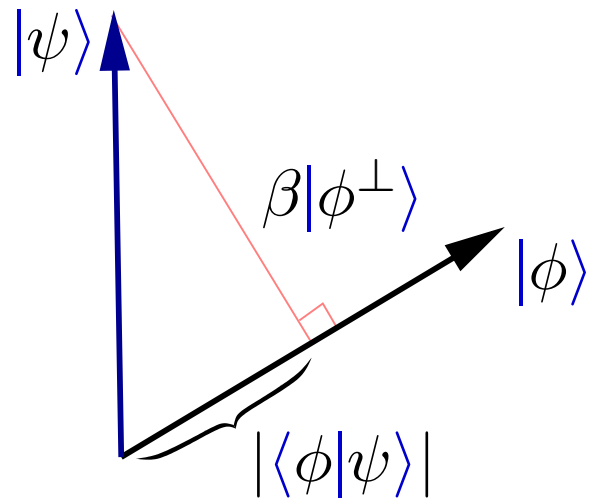
Overlap

- The overlap between $|\psi\rangle$ and $|\phi\rangle$ is $\langle\phi|\psi\rangle$.



Overlap

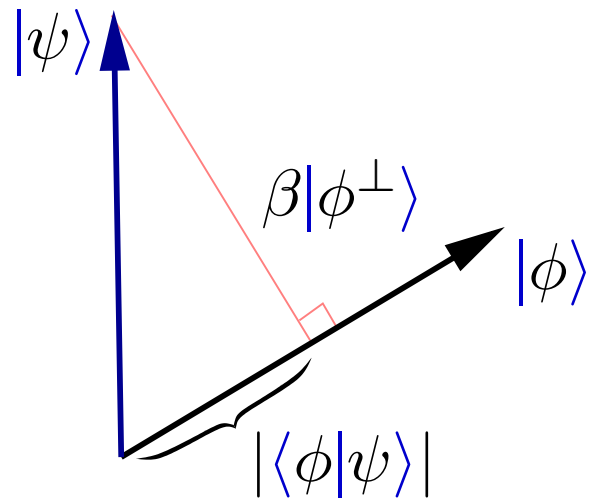
- The overlap between $|\psi\rangle$ and $|\phi\rangle$ is $\langle\phi|\psi\rangle$.



$$|\psi\rangle = \langle\phi|\psi\rangle|\phi\rangle + \beta|\phi^{\perp}\rangle$$

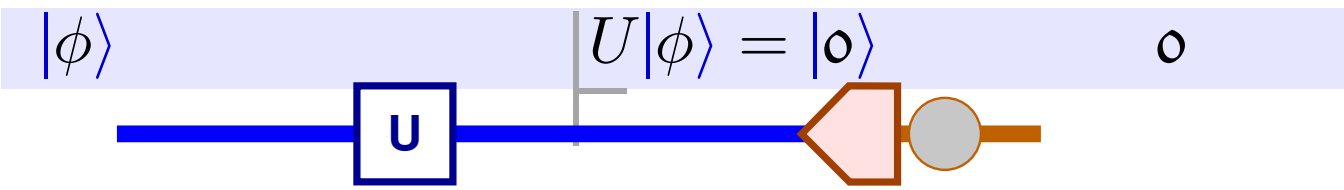
Overlap

- The overlap between $|\psi\rangle$ and $|\phi\rangle$ is $\langle\phi|\psi\rangle$.



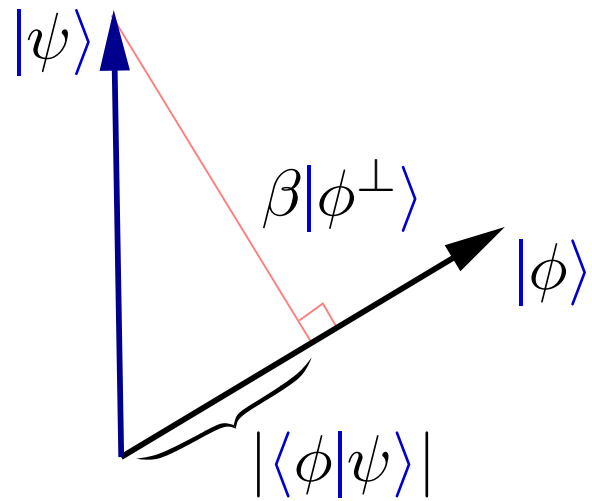
$$|\psi\rangle = \langle\phi|\psi\rangle|\phi\rangle + \beta|\phi^{\perp}\rangle$$

- Consider a measurement to determine if the state is $|\phi\rangle$.



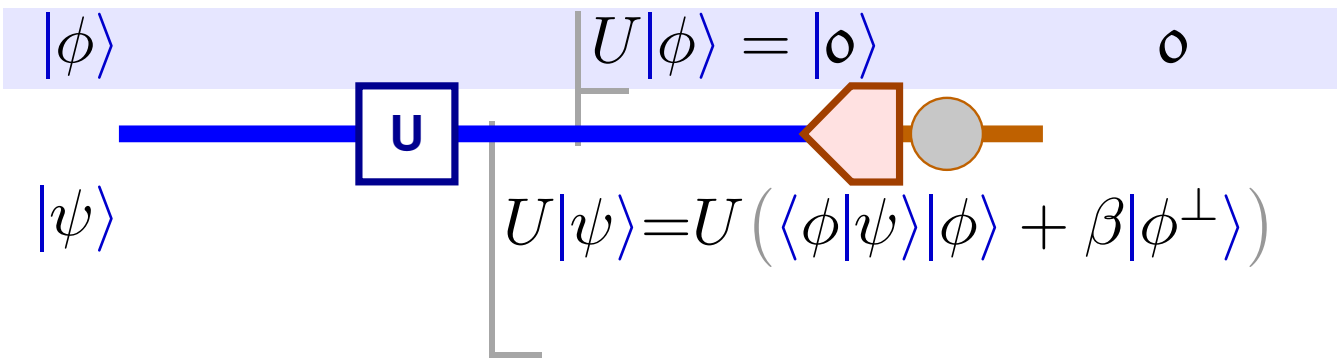
Overlap

- The overlap between $|\psi\rangle$ and $|\phi\rangle$ is $\langle\phi|\psi\rangle$.



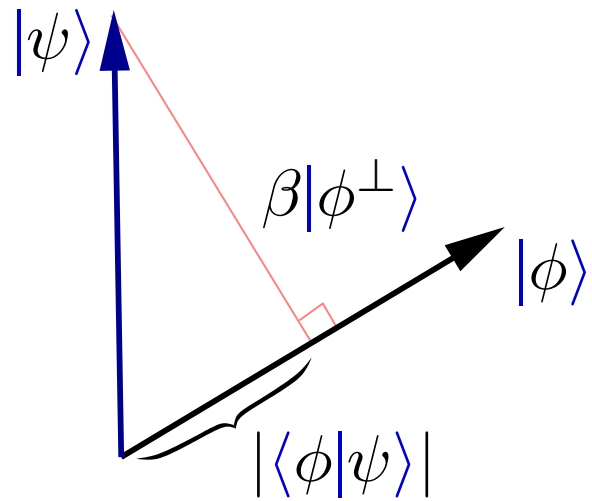
$$|\psi\rangle = \langle\phi|\psi\rangle|\phi\rangle + \beta|\phi^{\perp}\rangle$$

- Consider a measurement to determine if the state is $|\phi\rangle$.



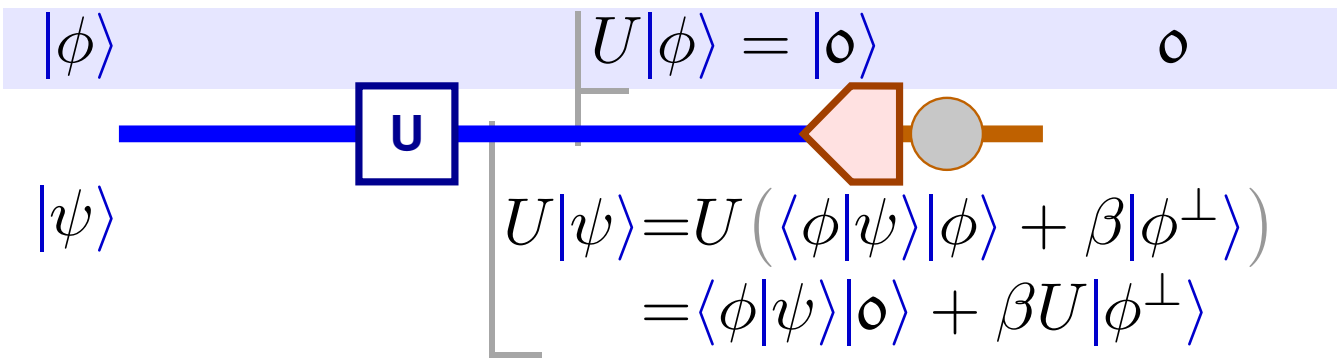
Overlap

- The overlap between $|\psi\rangle$ and $|\phi\rangle$ is $\langle\phi|\psi\rangle$.



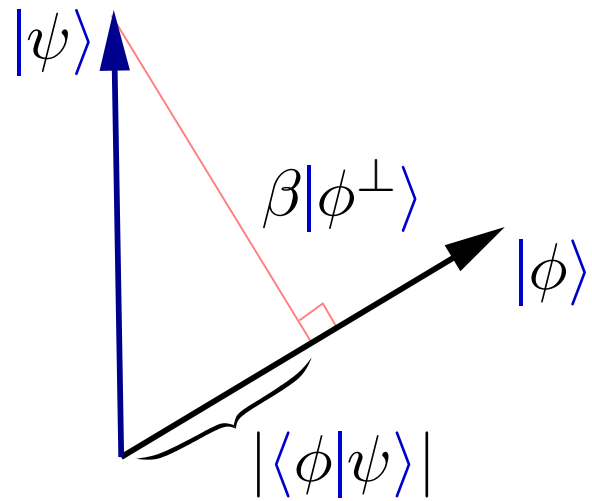
$$|\psi\rangle = \langle\phi|\psi\rangle|\phi\rangle + \beta|\phi^{\perp}\rangle$$

- Consider a measurement to determine if the state is $|\phi\rangle$.



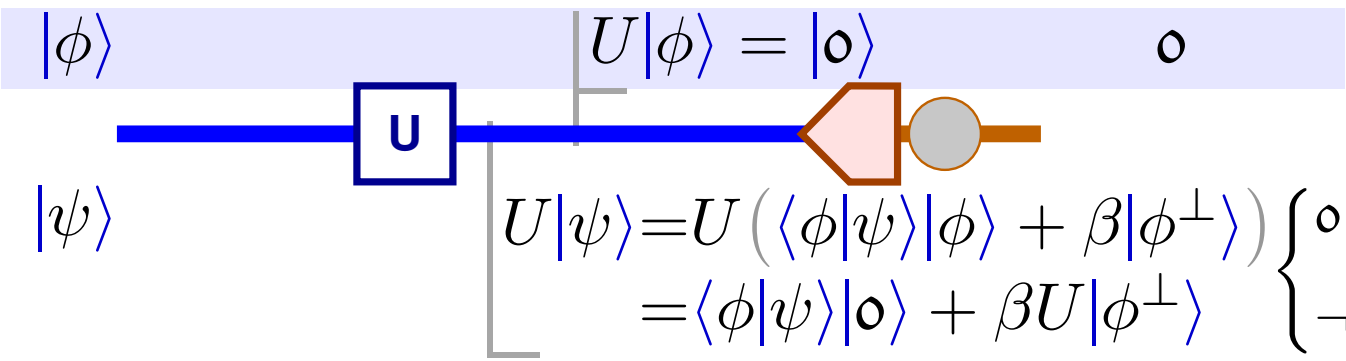
Overlap

- The overlap between $|\psi\rangle$ and $|\phi\rangle$ is $\langle\phi|\psi\rangle$.



$$|\psi\rangle = \langle\phi|\psi\rangle|\phi\rangle + \beta|\phi^{\perp}\rangle$$

- Consider a measurement to determine if the state is $|\phi\rangle$.

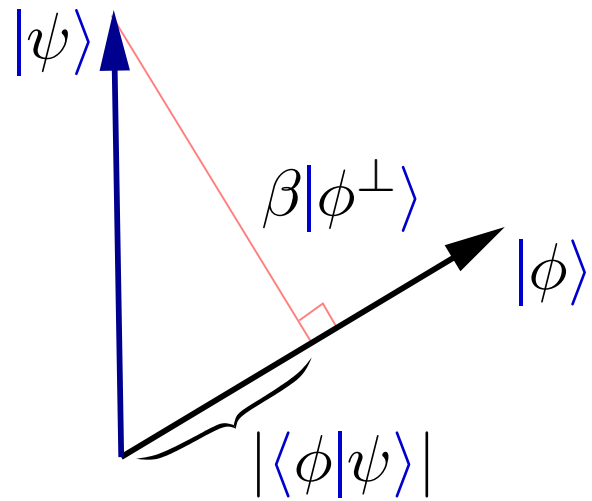


$$U|\psi\rangle = U\left(\langle\phi|\psi\rangle|\phi\rangle + \beta|\phi^{\perp}\rangle\right) = \langle\phi|\psi\rangle|0\rangle + \beta U|\phi^{\perp}\rangle$$

$$\begin{cases} 0: & \text{prob.} = |\langle\phi|\psi\rangle|^2 \\ \neg 0: & \text{prob.} = |\beta|^2 = 1 - |\langle\phi|\psi\rangle|^2 \end{cases}$$

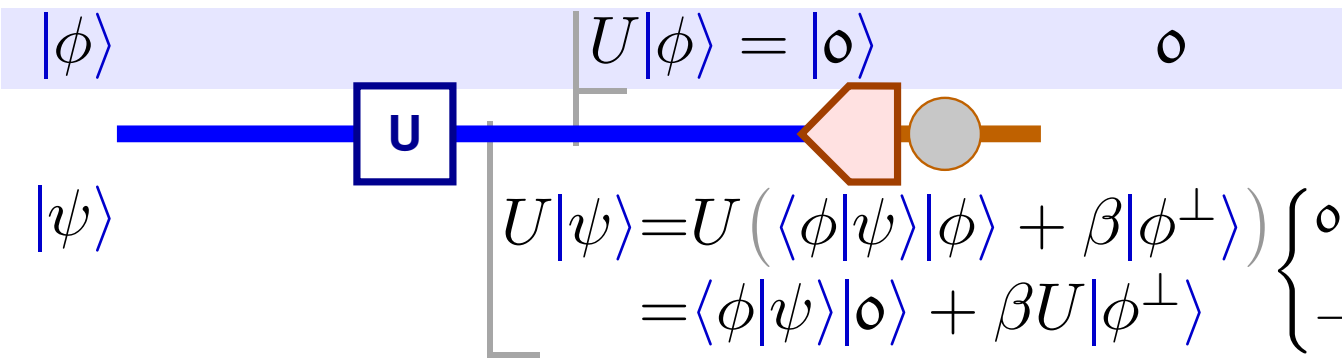
Overlap

- The overlap between $|\psi\rangle$ and $|\phi\rangle$ is $\langle\phi|\psi\rangle$.



$$|\psi\rangle = \langle\phi|\psi\rangle|\phi\rangle + \beta|\phi^{\perp}\rangle$$

- Consider a measurement to determine if the state is $|\phi\rangle$.

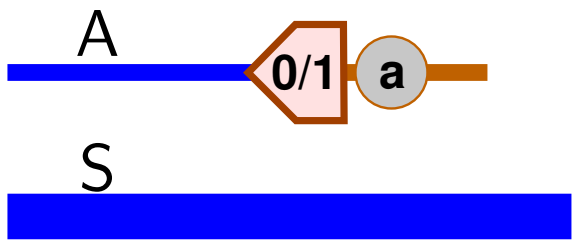


$$U|\psi\rangle = U\left(\langle\phi|\psi\rangle|\phi\rangle + \beta|\phi^{\perp}\rangle\right) = \langle\phi|\psi\rangle|0\rangle + \beta U|\phi^{\perp}\rangle$$

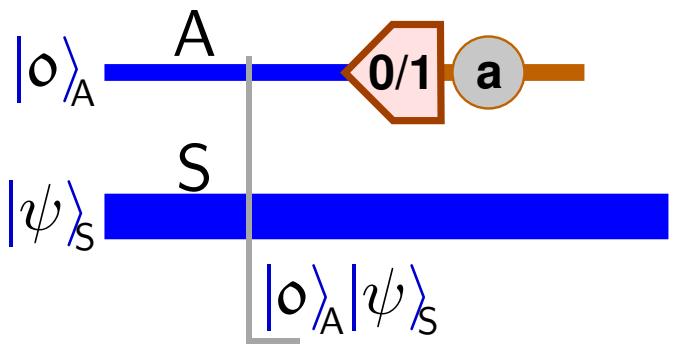
$$\begin{cases} 0: & \text{prob.} = |\langle\phi|\psi\rangle|^2 \\ \neg 0: & \text{prob.} = |\beta|^2 = 1 - |\langle\phi|\psi\rangle|^2 \end{cases}$$

- $|\text{overlap}|^2$ is the prob. of detecting $|\phi\rangle$ if the state is $|\psi\rangle$.

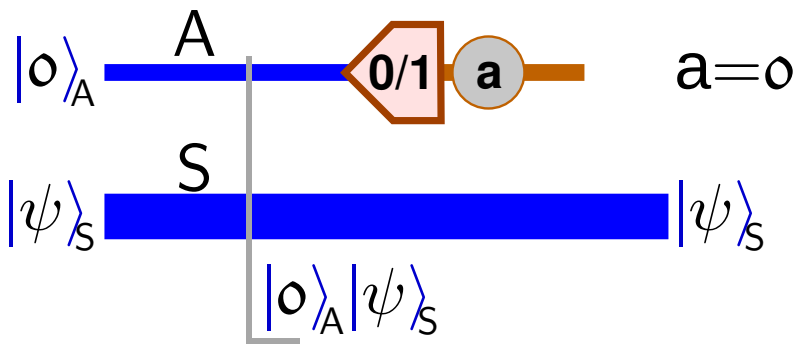
Measuring One Qubit Among Many



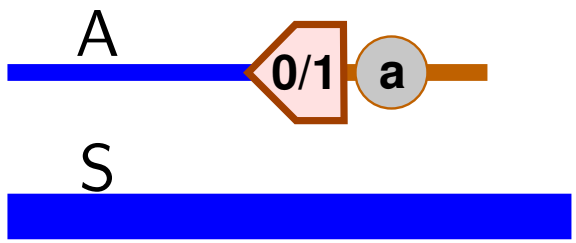
Measuring One Qubit Among Many



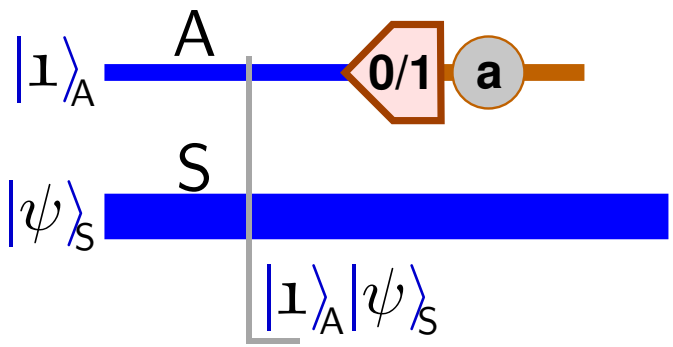
Measuring One Qubit Among Many



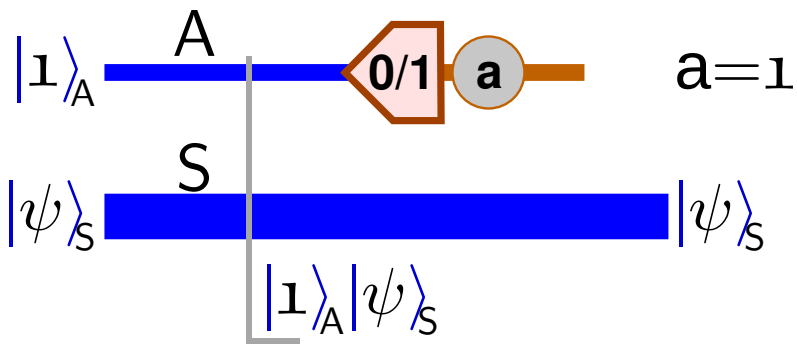
Measuring One Qubit Among Many



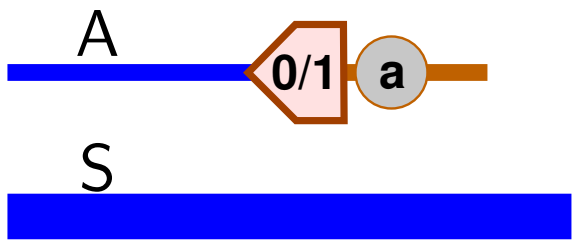
Measuring One Qubit Among Many



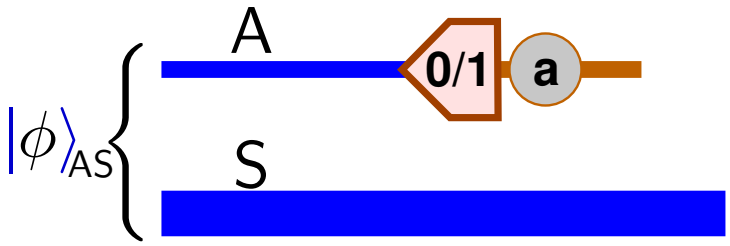
Measuring One Qubit Among Many



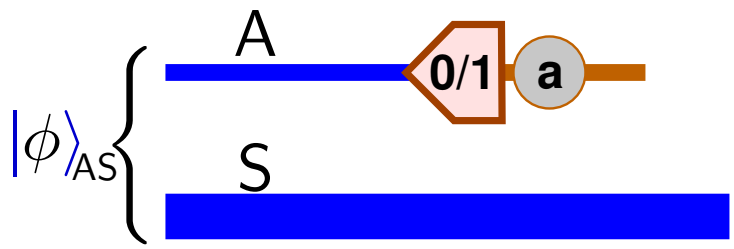
Measuring One Qubit Among Many



Measuring One Qubit Among Many



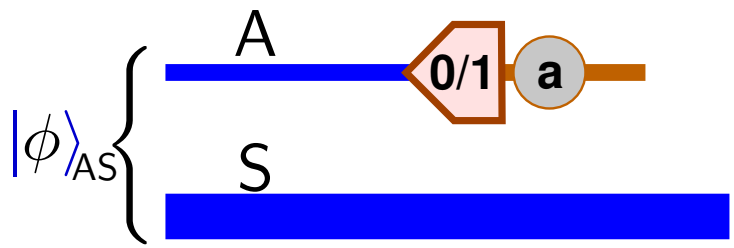
Measuring One Qubit Among Many



Write $|\phi\rangle_{AS} = |\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)$,

where $|\psi_0\rangle_S$ and $|\psi_1\rangle_S$ are states.

Measuring One Qubit Among Many

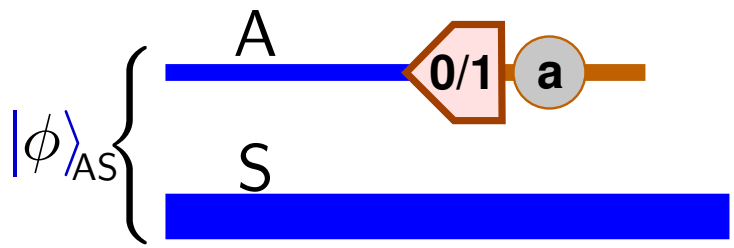


prob.$=|\alpha|^2$	prob.$=|\beta|^2$
a$=0$	a$=1$
$|\psi_0\rangle_S$	$|\psi_1\rangle_S$

Write $|\phi\rangle_{AS} = |0\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |1\rangle_A\big(\beta|\psi_1\rangle_S\big)$,

where $|\psi_0\rangle_S$ and $|\psi_1\rangle_S$ are states.

Measuring One Qubit Among Many



prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathtt{a}=\mathtt{0}$	$\mathtt{a}=\mathtt{1}$
$|\psi_0\rangle_S$	$|\psi_1\rangle_S$

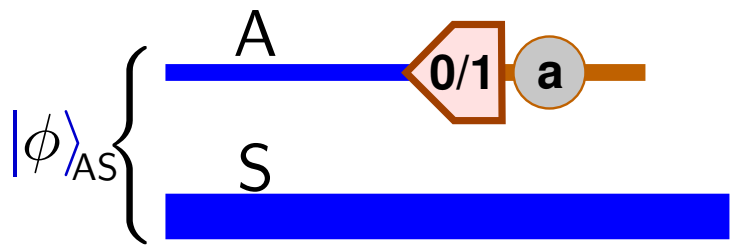
Write $|\phi\rangle_{AS} = |\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)$,

where $|\psi_0\rangle_S$ and $|\psi_1\rangle_S$ are states.

- Computing the probabilities.

$$\text{prob}(\mathtt{a} = \mathfrak{b}) \quad = \quad |{}^A\langle\mathfrak{b}||\phi\rangle_{AS}|^2$$

Measuring One Qubit Among Many



prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathtt{a}=\mathtt{0}$	$\mathtt{a}=\mathtt{1}$
$|\psi_0\rangle_S$	$|\psi_1\rangle_S$

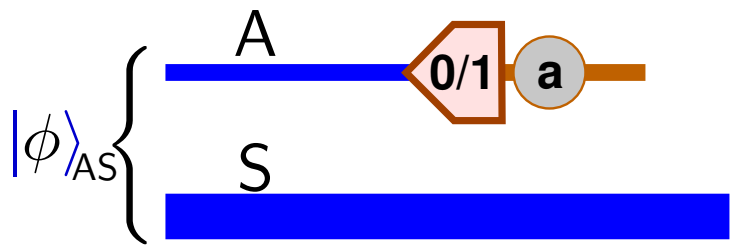
Write $|\phi\rangle_{AS} = |\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)$,

where $|\psi_0\rangle_S$ and $|\psi_1\rangle_S$ are states.

- Computing the probabilities.

$$\begin{aligned}\text{prob}(\mathtt{a}=\mathfrak{b}) &= |{}^{A}\langle\mathfrak{b}||\phi\rangle_{AS}|^2 \\ &= {}^{AS}\langle\phi||\mathfrak{b}\rangle_A\,{}^{A}\langle\mathfrak{b}||\phi\rangle_{AS}\end{aligned}$$

Measuring One Qubit Among Many



prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathtt{a}=\mathtt{0}$	$\mathtt{a}=\mathtt{1}$
$|\psi_0\rangle_S$	$|\psi_1\rangle_S$

Write $|\phi\rangle_{AS} = |\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)$,

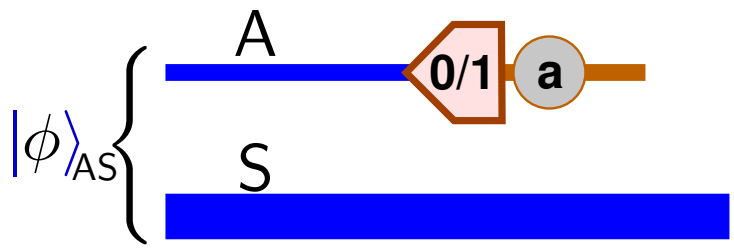
where $|\psi_0\rangle_S$ and $|\psi_1\rangle_S$ are states.

- Computing the probabilities.

$$\begin{aligned}\text{prob}(\mathtt{a}=\mathfrak{b}) &= |{}^{A}\langle\mathfrak{b}||\phi\rangle_{AS}|^2 \\ &= {}^{AS}\langle\phi||\mathfrak{b}\rangle_A\,{}^{A}\langle\mathfrak{b}||\phi\rangle_{AS}\end{aligned}$$

Check: $${}^{A}\langle\mathtt{0}||\phi\rangle_{AS} = {}^{A}\langle\mathtt{0}|\Big(|\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)\Big)$$

Measuring One Qubit Among Many



prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathtt{a}=\mathtt{0}$	$\mathtt{a}=\mathtt{1}$
$|\psi_0\rangle_S$	$|\psi_1\rangle_S$

Write $|\phi\rangle_{AS} = |\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)$,

where $|\psi_0\rangle_S$ and $|\psi_1\rangle_S$ are states.

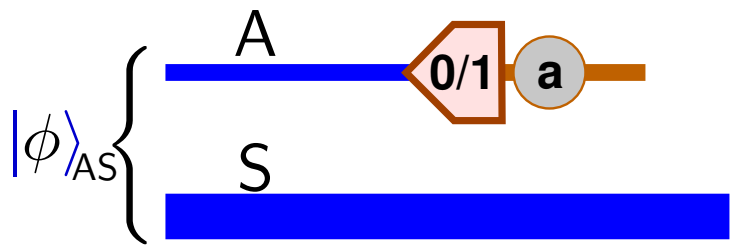
- Computing the probabilities.

$$\begin{aligned}\text{prob}(\mathtt{a}=\mathfrak{b}) &= |{}^{A}\langle\mathfrak{b}||\phi\rangle_{AS}|^2 \\ &= {}^{AS}\langle\phi||\mathfrak{b}\rangle_A\,{}^{A}\langle\mathfrak{b}||\phi\rangle_{AS}\end{aligned}$$

Check:

$$\begin{aligned}{}^{A}\langle\mathtt{0}||\phi\rangle_{AS} &= {}^{A}\langle\mathtt{0}|\Big(|\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)\Big) \\ &= {}^{A}\langle\mathtt{0}|\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + {}^{A}\langle\mathtt{0}|\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)\end{aligned}$$

Measuring One Qubit Among Many



prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathtt{a}=\mathtt{0}$	$\mathtt{a}=\mathtt{1}$
$|\psi_0\rangle_S$	$|\psi_1\rangle_S$

Write $|\phi\rangle_{AS} = |\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)$,

where $|\psi_0\rangle_S$ and $|\psi_1\rangle_S$ are states.

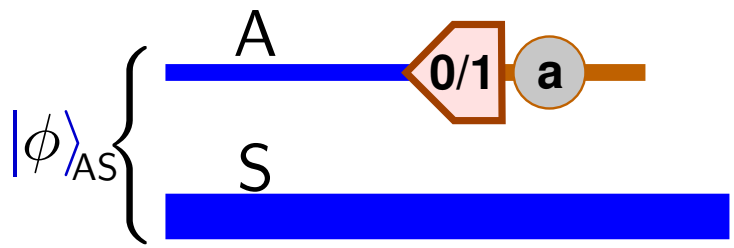
- Computing the probabilities.

$$\begin{aligned}\text{prob}(\mathtt{a}=\mathfrak{b}) &= |{}^A\langle\mathfrak{b}||\phi\rangle_{AS}|^2 \\ &= {}^{AS}\langle\phi||\mathfrak{b}\rangle_A\,{}^A\langle\mathfrak{b}||\phi\rangle_{AS}\end{aligned}$$

Check:

$$\begin{aligned}{}^A\langle\mathtt{0}||\phi\rangle_{AS} &= {}^A\langle\mathtt{0}|\Big(|\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)\Big) \\ &= \underbrace{{}^A\langle\mathtt{0}|\mathtt{0}\rangle_A}_{1}\big(\alpha|\psi_0\rangle_S\big) + \underbrace{{}^A\langle\mathtt{0}|\mathtt{1}\rangle_A}_{0}\big(\beta|\psi_1\rangle_S\big)\end{aligned}$$

Measuring One Qubit Among Many



prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathtt{a}=\mathtt{0}$	$\mathtt{a}=\mathtt{1}$
$|\psi_0\rangle_S$	$|\psi_1\rangle_S$

Write $|\phi\rangle_{AS} = |\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)$,

where $|\psi_0\rangle_S$ and $|\psi_1\rangle_S$ are states.

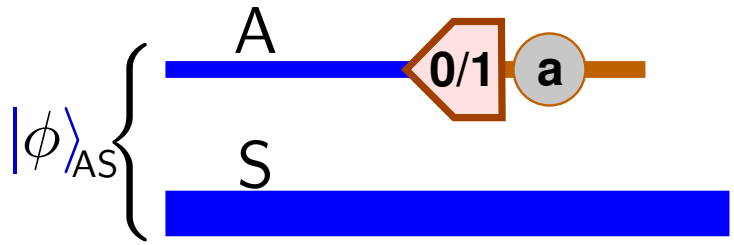
- Computing the probabilities.

$$\begin{aligned}
\text{prob}(\mathtt{a}=\mathfrak{b}) &= |{}^A\langle\mathfrak{b}||\phi\rangle_{AS}|^2 \\
&= {}^{AS}\langle\phi||\mathfrak{b}\rangle_A\,{}^A\langle\mathfrak{b}||\phi\rangle_{AS}
\end{aligned}$$

Check:

$$\begin{aligned}
{}^A\langle\mathtt{0}||\phi\rangle_{AS} &= {}^A\langle\mathtt{0}|\Big(|\mathtt{0}\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |\mathtt{1}\rangle_A\big(\beta|\psi_1\rangle_S\big)\Big) \\
&= \underbrace{{}^A\langle\mathtt{0}|\mathtt{0}\rangle_A}_{1}\big(\alpha|\psi_0\rangle_S\big) + \underbrace{{}^A\langle\mathtt{0}|\mathtt{1}\rangle_A}_{0}\big(\beta|\psi_1\rangle_S\big) \\
&= \alpha|\psi_0\rangle_S
\end{aligned}$$

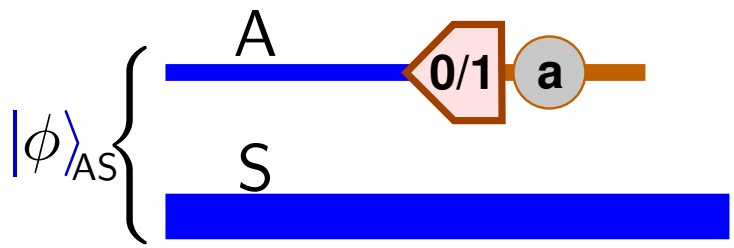
Measuring One Qubit Among Many



prob.$=|\alpha|^2$	prob.$=|\beta|^2$
a$=0$	a$=1$
$|\psi_0\rangle_S$	$|\psi_1\rangle_S$

$$|\phi\rangle_{AS} = |0\rangle_A\big(\alpha|\psi_0\rangle_S\big) + |1\rangle_A\big(\beta|\psi_1\rangle_S\big)$$

Measuring One Qubit Among Many



$|\phi\rangle_{\mathsf{AS}}$

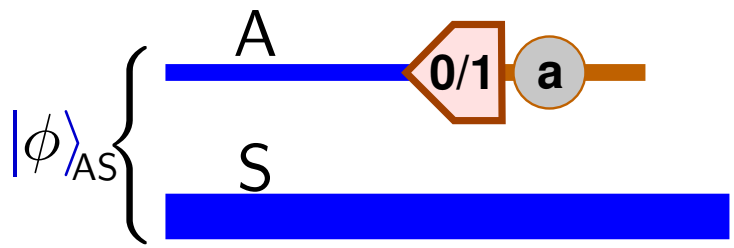
prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathtt{a}=\mathtt{0}$	$\mathtt{a}=\mathtt{1}$
$|\psi_0\rangle_{\mathsf{S}}$	$|\psi_1\rangle_{\mathsf{S}}$

$$|\phi\rangle_{\mathsf{AS}} = |\mathtt{0}\rangle_{\mathsf{A}}\big(\alpha|\psi_0\rangle_{\mathsf{S}}\big) + |\mathtt{1}\rangle_{\mathsf{A}}\big(\beta|\psi_1\rangle_{\mathsf{S}}\big)$$

- Bra-ket expression for measurement?

$$\mathbf{meas}(Z\mapsto\mathsf{a}) = (\mathtt{0}\mapsto\mathsf{a})\,{}^{\mathsf{A}}\langle\mathtt{0}| + (\mathtt{1}\mapsto\mathsf{a})\,{}^{\mathsf{A}}\langle\mathtt{1}|$$

Measuring One Qubit Among Many



$|\phi\rangle_{\mathsf{AS}}$

prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathsf{a}=\mathtt{0}$	$\mathsf{a}=\mathtt{1}$
$|\psi_0\rangle_{\mathsf{S}}$	$|\psi_1\rangle_{\mathsf{S}}$

$$|\phi\rangle_{\mathsf{AS}} = |\mathtt{0}\rangle_{\mathsf{A}}\big(\alpha|\psi_0\rangle_{\mathsf{S}}\big) + |\mathtt{1}\rangle_{\mathsf{A}}\big(\beta|\psi_1\rangle_{\mathsf{S}}\big)$$

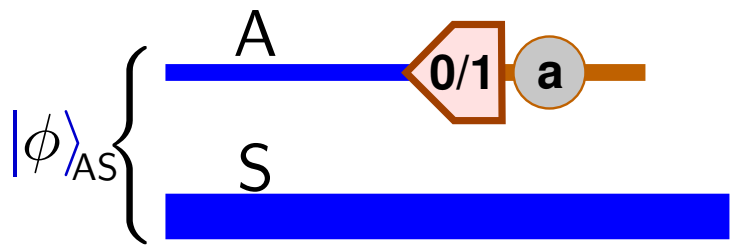
- Bra-ket expression for measurement?

$$\mathbf{meas}(Z\mapsto\mathsf{a}) = (\mathtt{0}\mapsto\mathsf{a})\,{}^{\mathsf{A}}\langle\mathtt{0}| + (\mathtt{1}\mapsto\mathsf{a})\,{}^{\mathsf{A}}\langle\mathtt{1}|$$

  - Effect when applied to $|\phi\rangle_{\mathsf{AS}}$:

$$\mathbf{meas}(Z\mapsto\mathsf{a})|\phi\rangle_{\mathsf{AS}} = (\mathtt{0}\mapsto\mathsf{a})\alpha|\psi_0\rangle_{\mathsf{S}} + (\mathtt{1}\mapsto\mathsf{a})\beta|\psi_1\rangle_{\mathsf{S}}.$$

Measuring One Qubit Among Many



$|\phi\rangle_{\mathsf{AS}}$

prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathsf{a}=\mathtt{0}$	$\mathsf{a}=\mathtt{1}$
$|\psi_0\rangle_{\mathsf{S}}$	$|\psi_1\rangle_{\mathsf{S}}$

$$|\phi\rangle_{\mathsf{AS}} = |\mathtt{0}\rangle_{\mathsf{A}}\big(\alpha|\psi_0\rangle_{\mathsf{S}}\big) + |\mathtt{1}\rangle_{\mathsf{A}}\big(\beta|\psi_1\rangle_{\mathsf{S}}\big)$$

- Bra-ket expression for measurement?

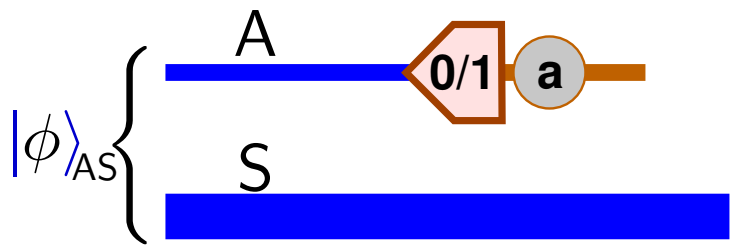
$$\mathbf{meas}(Z\mapsto\mathsf{a}) = (\mathtt{0}\mapsto\mathsf{a})\,{}^{\mathsf{A}}\langle\mathtt{0}| + (\mathtt{1}\mapsto\mathsf{a})\,{}^{\mathsf{A}}\langle\mathtt{1}|$$

  - Effect when applied to $|\phi\rangle_{\mathsf{AS}}$:

$$\mathbf{meas}(Z\mapsto\mathsf{a})|\phi\rangle_{\mathsf{AS}} = (\mathtt{0}\mapsto\mathsf{a})\alpha|\psi_0\rangle_{\mathsf{S}} + (\mathtt{1}\mapsto\mathsf{a})\beta|\psi_1\rangle_{\mathsf{S}}.$$

...classically labeled sum of unnormalized states.

Measuring One Qubit Among Many



$|\phi\rangle_{\mathsf{AS}}$

prob.$=|\alpha|^2$	prob.$=|\beta|^2$
$\mathsf{a}=\mathtt{0}$	$\mathsf{a}=\mathtt{1}$
$|\psi_0\rangle_{\mathsf{S}}$	$|\psi_1\rangle_{\mathsf{S}}$

$$|\phi\rangle_{\mathsf{AS}} = |\mathtt{0}\rangle_{\mathsf{A}}\big(\alpha|\psi_0\rangle_{\mathsf{S}}\big) + |\mathtt{1}\rangle_{\mathsf{A}}\big(\beta|\psi_1\rangle_{\mathsf{S}}\big)$$

- Bra-ket expression for measurement?

$$\mathbf{meas}(Z\mapsto\mathsf{a}) = (\mathtt{0}\mapsto\mathsf{a})\,{}^{\mathsf{A}}\langle\mathtt{0}| + (\mathtt{1}\mapsto\mathsf{a})\,{}^{\mathsf{A}}\langle\mathtt{1}|$$

  - Effect when applied to $|\phi\rangle_{\mathsf{AS}}$:

$$\mathbf{meas}(Z\mapsto\mathsf{a})|\phi\rangle_{\mathsf{AS}} = (\mathtt{0}\mapsto\mathsf{a})\alpha|\psi_0\rangle_{\mathsf{S}} + (\mathtt{1}\mapsto\mathsf{a})\beta|\psi_1\rangle_{\mathsf{S}}.$$

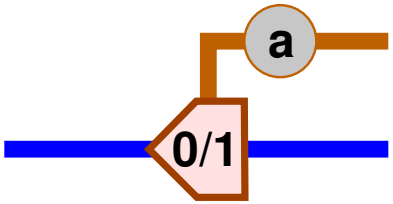
...classically labeled sum of unnormalized states.

- Unnormalized state conventions. Consider $|\omega\rangle$ with $\langle\omega|\omega\rangle \neq 1$.
  1. The intended state is $\frac{1}{\sqrt{\langle\omega|\omega\rangle}}|\omega\rangle$
  2. Amplitude meaningful? Then the state occurs with probability $\langle\omega|\omega\rangle$.

Projective Measurement

- Projective or Von Neumann measurement of σ_z.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

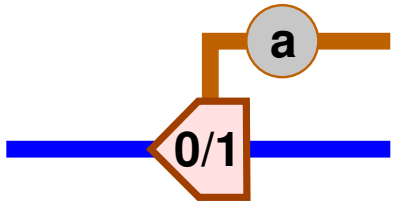


prob.$=\|\alpha\|^2$	prob.$=\|\beta\|^2$
a$=0$	a$=1$
$\|0\rangle\propto$	$\|1\rangle\propto$
$\|0\rangle\langle 0\|\psi\rangle$	$\|1\rangle\langle 1\|\psi\rangle$

Projective Measurement

- Projective or Von Neumann measurement of σ_z.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



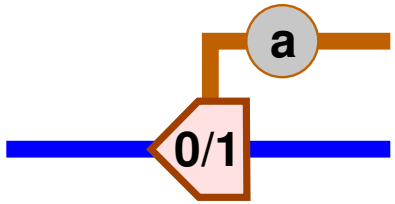
prob.$=|\alpha|^2$	prob.$=|\beta|^2$
a$=0$	a$=1$
$|0\rangle \propto$	$|1\rangle \propto$
$|0\rangle\langle 0||\psi\rangle$	$|1\rangle\langle 1||\psi\rangle$

  - $P_1 = |0\rangle\langle 0|$, $P_2 = |1\rangle\langle 1|$: Measurement projection operators.

Projective Measurement

- Projective or Von Neumann measurement of σ_z.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



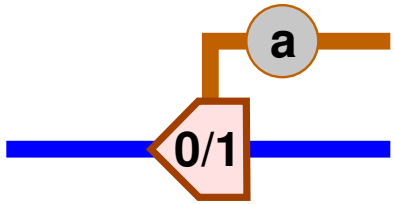
prob.$=|\alpha|^2$	prob.$=|\beta|^2$
a$=0$	a$=1$
$|0\rangle \propto$	$|1\rangle \propto$
$|0\rangle\langle 0||\psi\rangle$	$|1\rangle\langle 1||\psi\rangle$

  - $P_1 = |0\rangle\langle 0|$, $P_2 = |1\rangle\langle 1|$: Measurement projection operators.
    $$P_i = P_i^\dagger, P_i^2 = P_i, P_1 + P_2 = \mathbb{1}.$$

Projective Measurement

- Projective or Von Neumann measurement of σ_z.

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

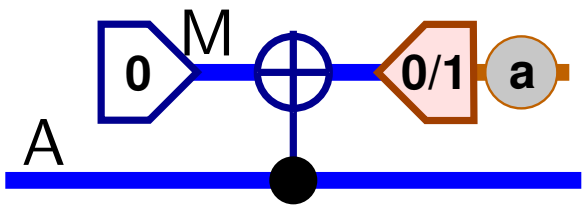


prob.$=|\alpha|^2$	prob.$=|\beta|^2$
a$=0$	a$=1$
$|0\rangle\propto$	$|1\rangle\propto$
$|0\rangle\langle 0||\psi\rangle$	$|1\rangle\langle 1||\psi\rangle$

  - $P_1 = |0\rangle\langle 0|$, $P_2 = |1\rangle\langle 1|$: Measurement projection operators.
    $$P_i = P_i^\dagger, P_i^2 = P_i, P_1 + P_2 = \mathbb{1}.$$

- Implementation with destructive measurement.

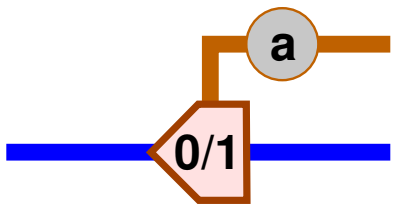
$\alpha|0\rangle_A + \beta|1\rangle_A$



Projective Measurement

- Projective or Von Neumann measurement of σ_z.

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

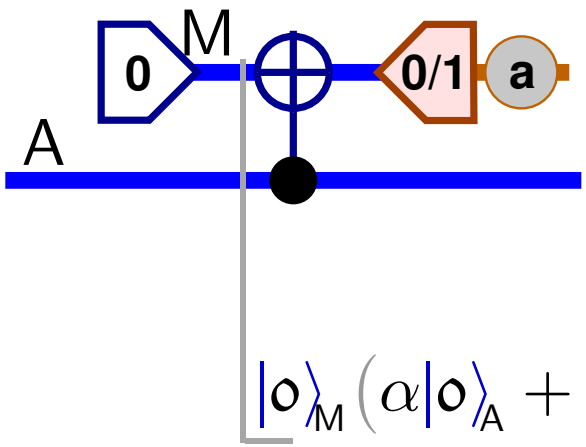


prob.$=|\alpha|^2$	prob.$=|\beta|^2$
a$=0$	a$=1$
$|0\rangle \propto$	$|1\rangle \propto$
$|0\rangle\langle 0||\psi\rangle$	$|1\rangle\langle 1||\psi\rangle$

  - $P_1 = |0\rangle\langle 0|$, $P_2 = |1\rangle\langle 1|$: Measurement projection operators.
    $$P_i = P_i^\dagger, P_i^2 = P_i, P_1 + P_2 = \mathbb{1}.$$

- Implementation with destructive measurement.

$\alpha|0\rangle_A + \beta|1\rangle_A$

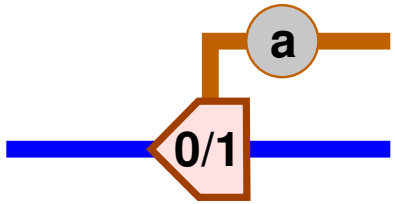


$|0\rangle_M\left(\alpha|0\rangle_A + \beta|1\rangle_A\right)$

Projective Measurement

- Projective or Von Neumann measurement of σ_z.

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

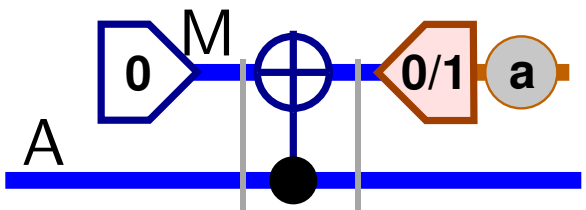


prob.$=|\alpha|^2$	prob.$=|\beta|^2$
a$=0$	a$=1$
$|0\rangle \propto$	$|1\rangle \propto$
$|0\rangle\langle 0||\psi\rangle$	$|1\rangle\langle 1||\psi\rangle$

  - $P_1 = |0\rangle\langle 0|$, $P_2 = |1\rangle\langle 1|$: Measurement projection operators.
    $$P_i = P_i^\dagger, P_i^2 = P_i, P_1 + P_2 = \mathbb{1}.$$

- Implementation with destructive measurement.

$\alpha|0\rangle_A + \beta|1\rangle_A$



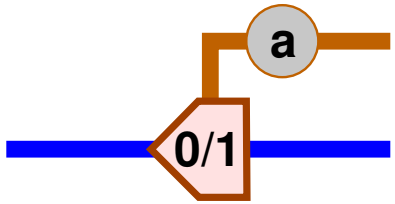
$\alpha|0\rangle_M|0\rangle_A + \beta|1\rangle_M|1\rangle_A$

$|0\rangle_M\left(\alpha|0\rangle_A + \beta|1\rangle_A\right)$

Projective Measurement

- Projective or Von Neumann measurement of σ_z.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

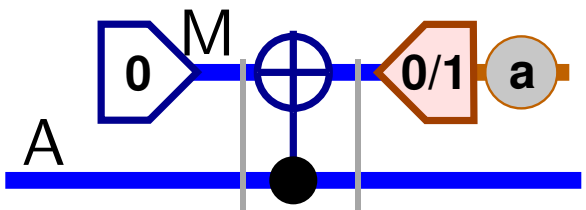


prob.$=\lvert\alpha\rvert^2$	prob.$=\lvert\beta\rvert^2$
a$=0$	a$=1$
$\lvert 0\rangle\propto$	$\lvert 1\rangle\propto$
$\lvert 0\rangle\langle 0\rvert\psi\rangle$	$\lvert 1\rangle\langle 1\rvert\psi\rangle$

  - $P_1 = |0\rangle\langle 0|$, $P_2 = |1\rangle\langle 1|$: Measurement projection operators.
    $$P_i = P_i^\dagger, P_i^2 = P_i, P_1 + P_2 = \mathbb{1}.$$

- Implementation with destructive measurement.

$$\alpha|0\rangle_A + \beta|1\rangle_A$$



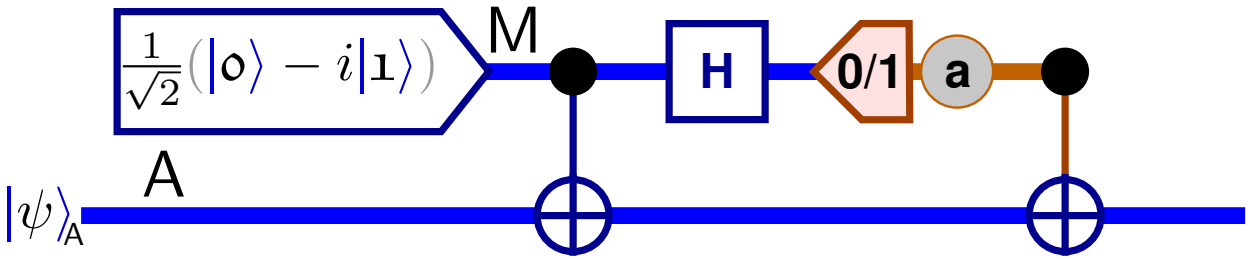
$$\alpha|0\rangle_M|0\rangle_A + \beta|1\rangle_M|1\rangle_A$$

$$|0\rangle_M\left(\alpha|0\rangle_A + \beta|1\rangle_A\right)$$

prob.$=\lvert\alpha\rvert^2$	prob.$=\lvert\beta\rvert^2$
a$=0$	a$=1$
$\lvert 0\rangle$	$\lvert 1\rangle$

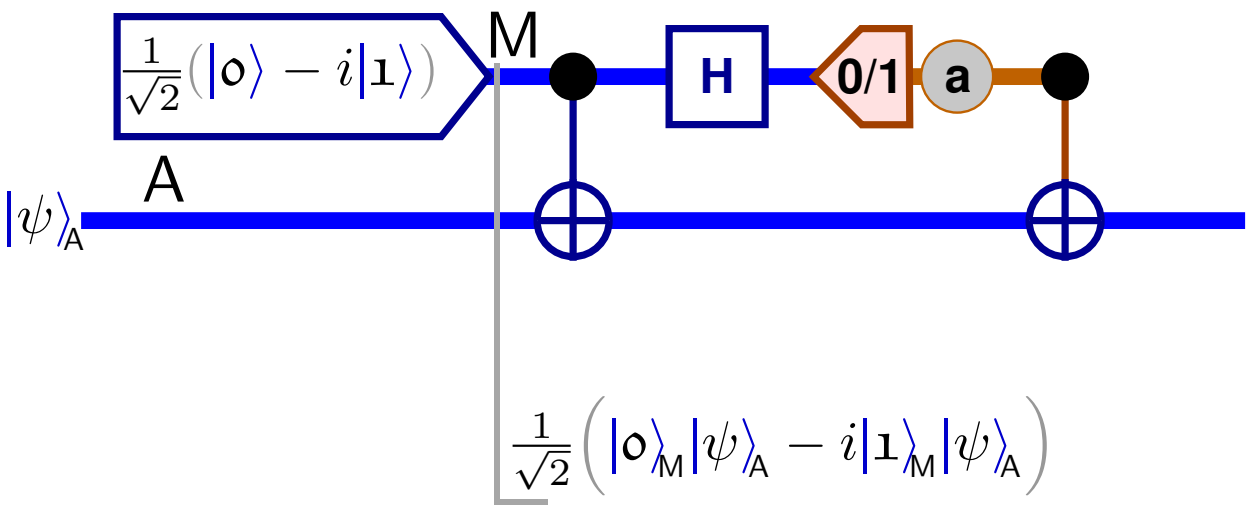
Rotations by State Preparation and Measurement

- $\mathbf{X}_{90^\circ}$ using $|i\pi/4\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)$.



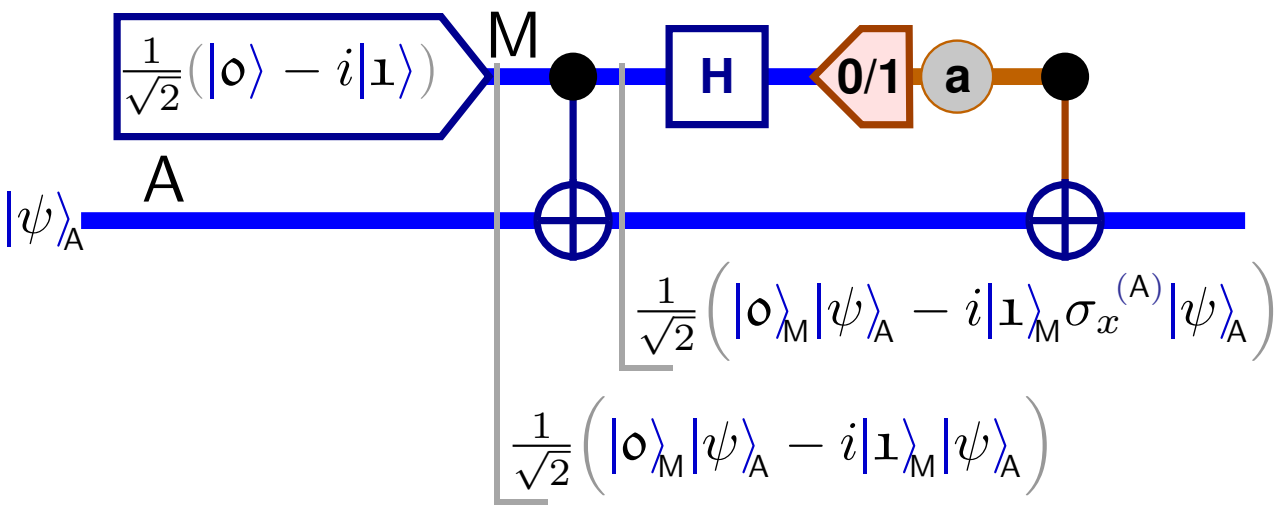
Rotations by State Preparation and Measurement

- $\mathbf{X}_{90^\circ}$ using $|i\pi/4\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)$.



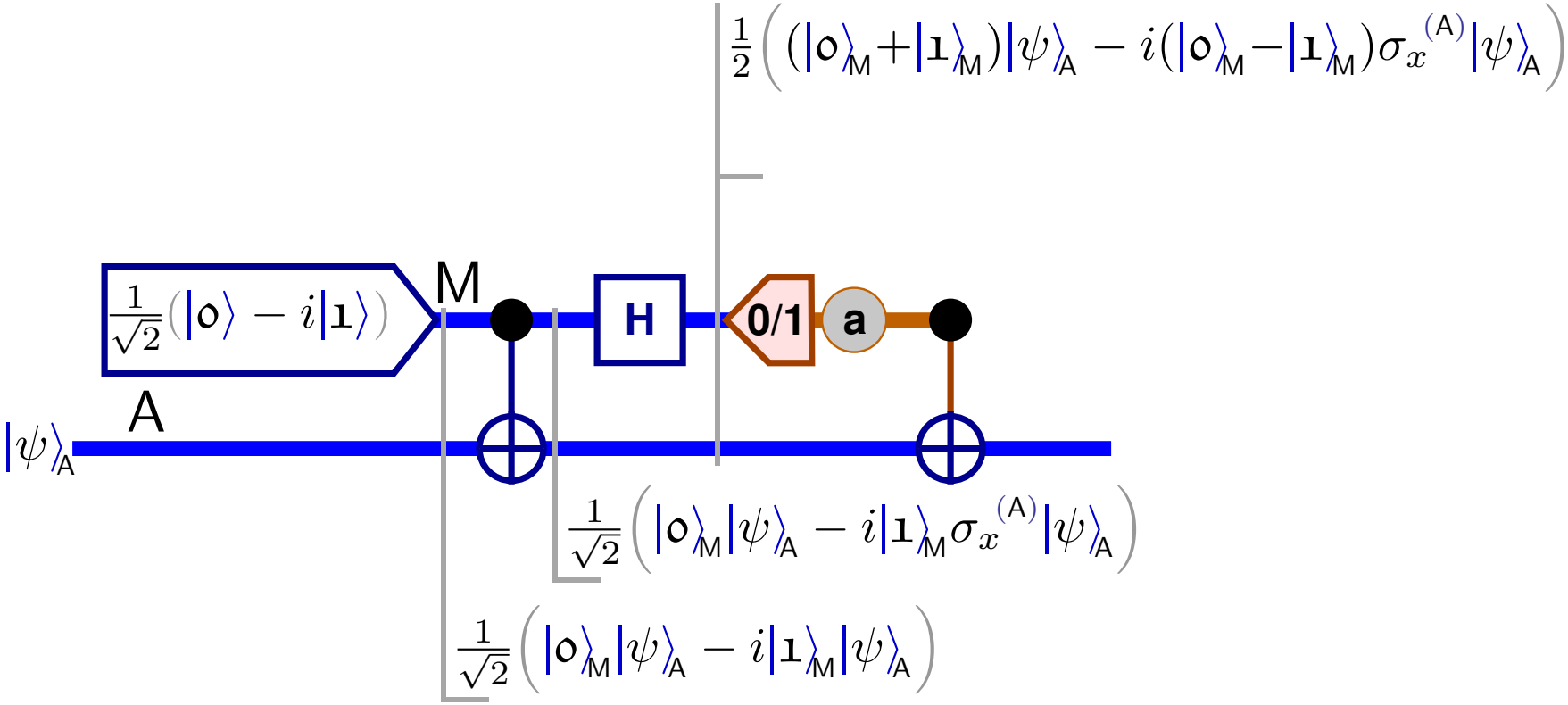
Rotations by State Preparation and Measurement

- $\mathbf{X}_{90^\circ}$ using $|i\pi/4\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)$.



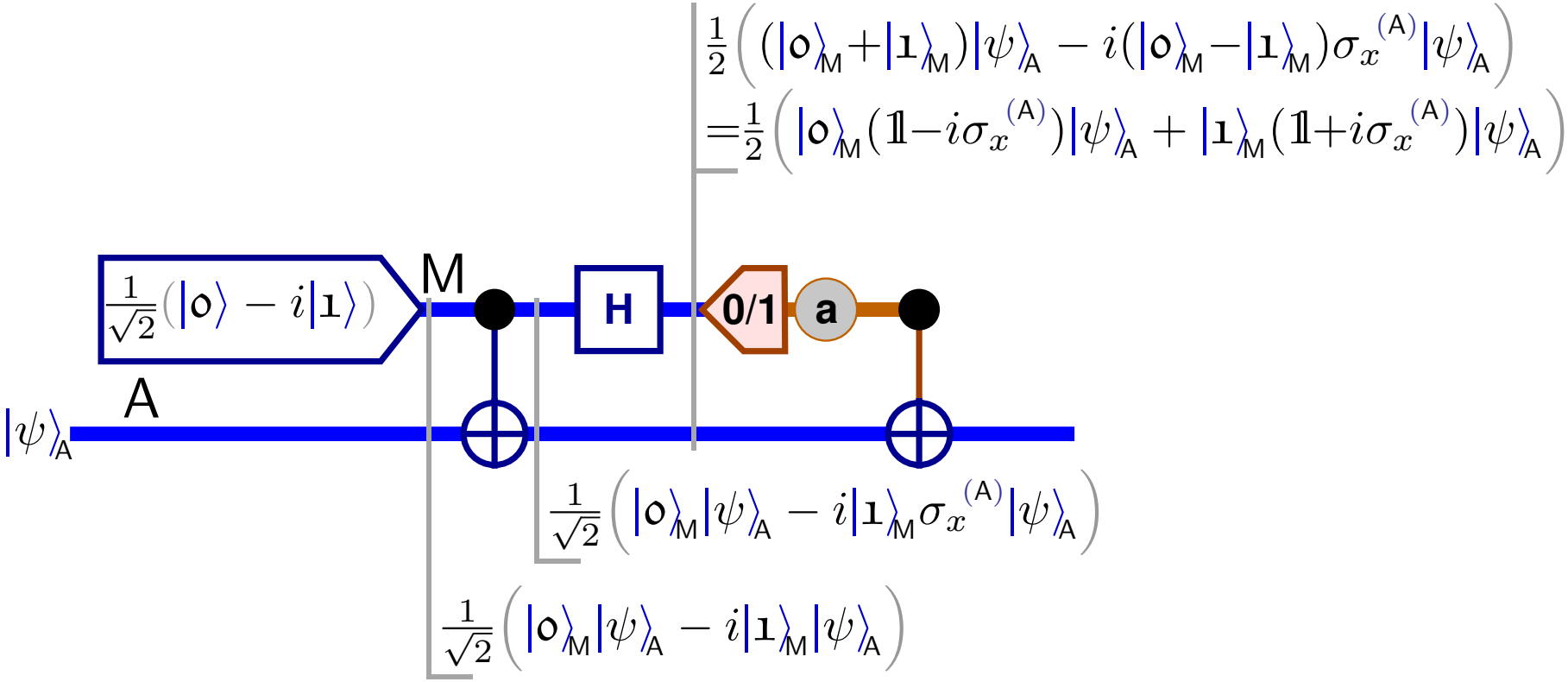
Rotations by State Preparation and Measurement

- $\mathbf{X}_{90^\circ}$ using $|i\pi/4\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)$.



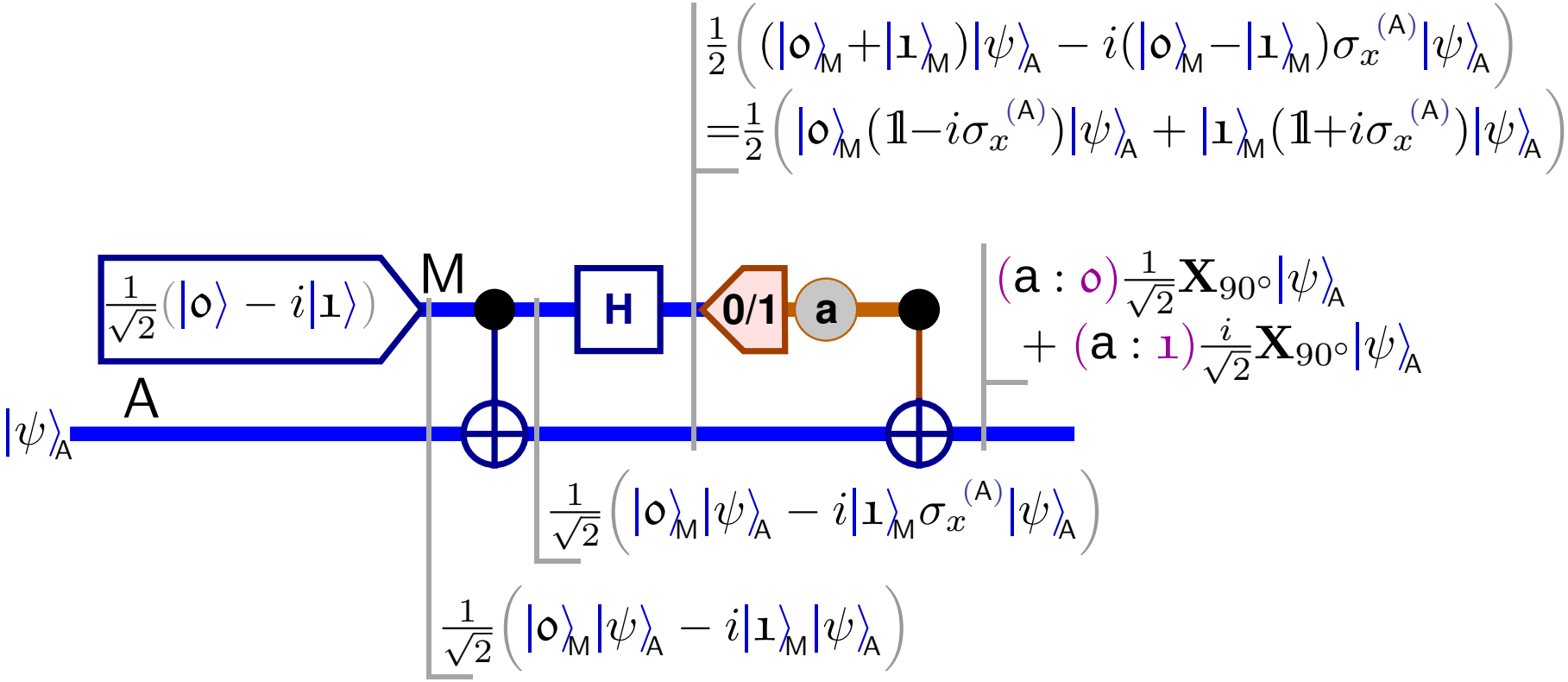
Rotations by State Preparation and Measurement

- $\mathbf{X}_{90^\circ}$ using $|i\pi/4\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)$.



Rotations by State Preparation and Measurement

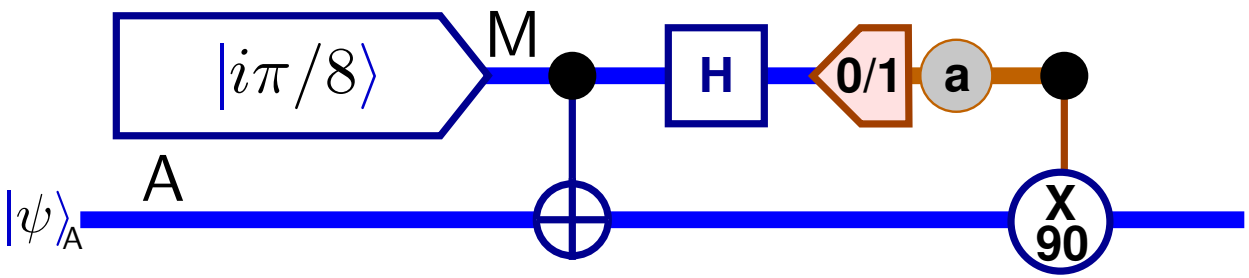
- $\mathbf{X}_{90^\circ}$ using $|i\pi/4\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)$.



Rotations by State Preparation and Measurement

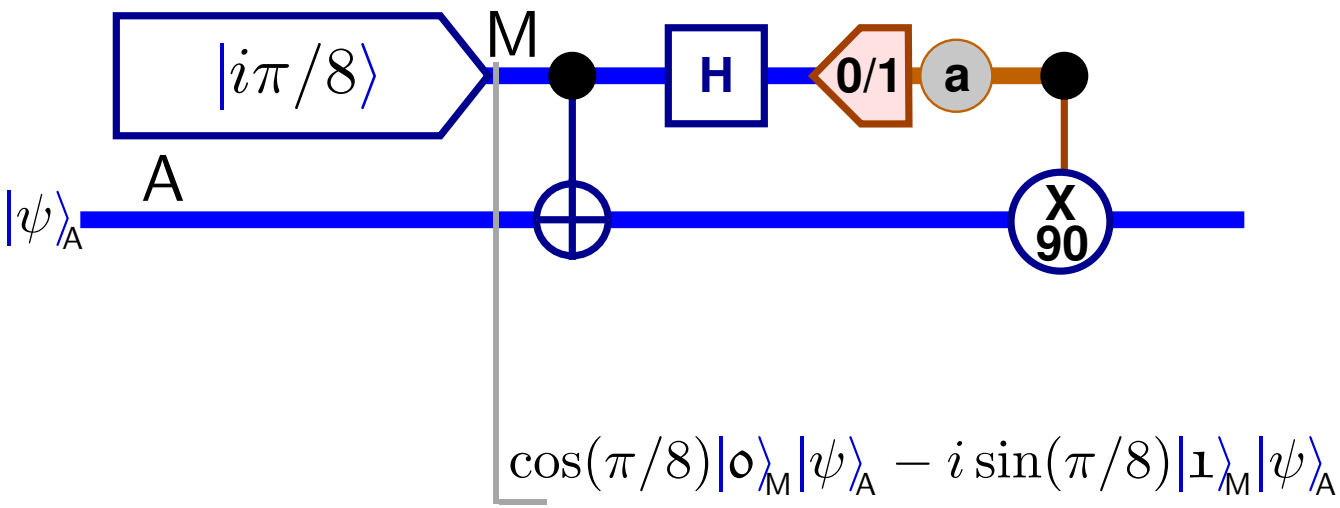
Rotations by State Preparation and Measurement

- $\mathbf{X}_{45^\circ}$ using $|i\pi/8\rangle = \cos(\pi/8)|0\rangle - i\sin(\pi/8)|1\rangle$.



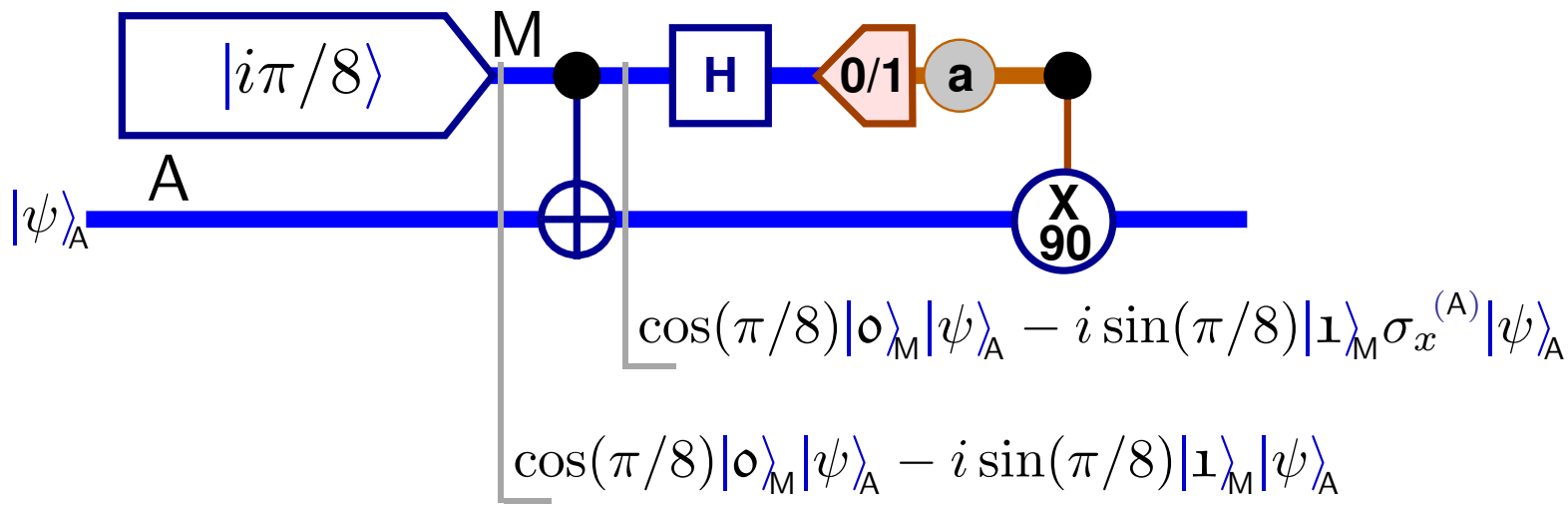
Rotations by State Preparation and Measurement

- $\mathbf{X}_{45^\circ}$ using $|i\pi/8\rangle = \cos(\pi/8)|\mathtt{0}\rangle - i\sin(\pi/8)|\mathtt{1}\rangle$.



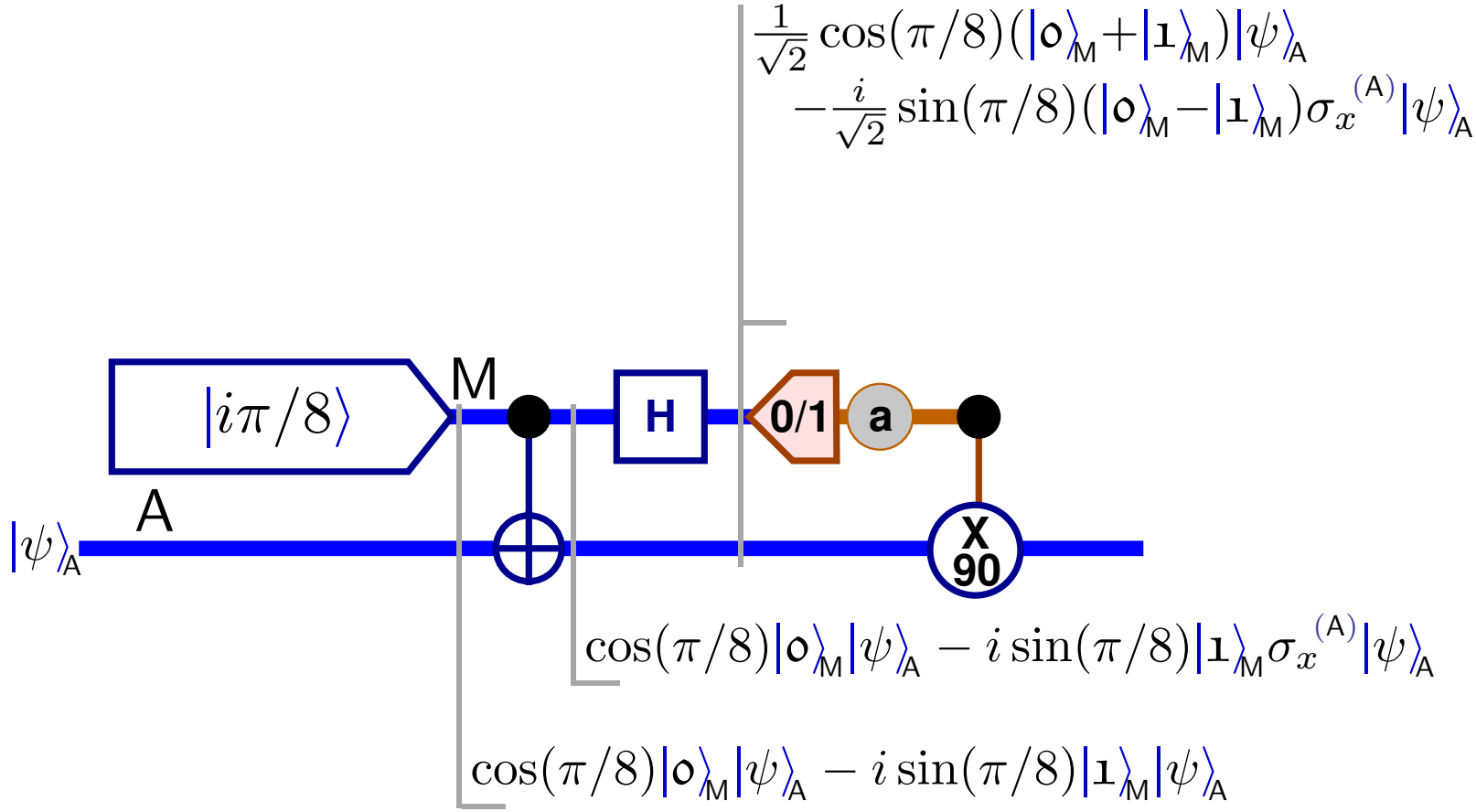
Rotations by State Preparation and Measurement

- $\mathbf{X}_{45^\circ}$ using $|i\pi/8\rangle = \cos(\pi/8)|\mathtt{0}\rangle - i\sin(\pi/8)|\mathtt{1}\rangle$.



Rotations by State Preparation and Measurement

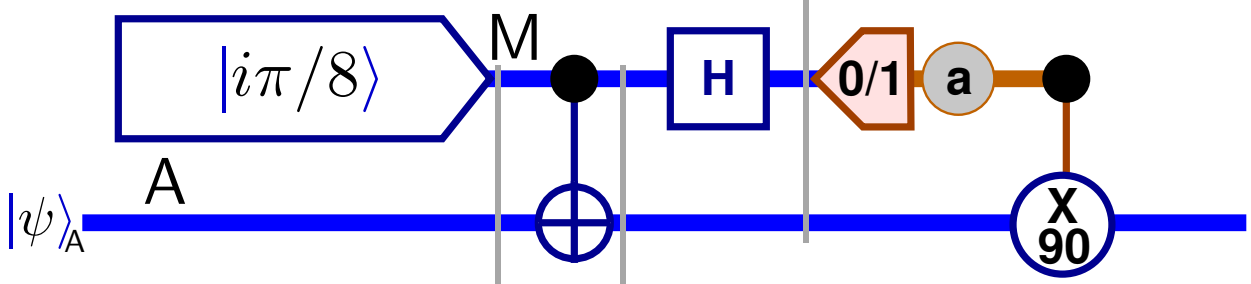
- $\mathbf{X}_{45^\circ}$ using $|i\pi/8\rangle = \cos(\pi/8)|0\rangle - i\sin(\pi/8)|1\rangle$.



Rotations by State Preparation and Measurement

- $\mathbf{X}_{45^\circ}$ using $|i\pi/8\rangle = \cos(\pi/8)|0\rangle - i\sin(\pi/8)|1\rangle$.

$$\frac{1}{\sqrt{2}}\cos(\pi/8)(|0\rangle_{\mathsf{M}}+|1\rangle_{\mathsf{M}})|\psi\rangle_{\mathsf{A}}$$
$$-\frac{i}{\sqrt{2}}\sin(\pi/8)(|0\rangle_{\mathsf{M}}-|1\rangle_{\mathsf{M}})\sigma_x^{(\mathsf{A})}|\psi\rangle_{\mathsf{A}}$$
$$=\frac{1}{\sqrt{2}}|0\rangle_{\mathsf{M}}(\cos(\pi/8)\mathbb{1}-i\sin(\pi/8)\sigma_x^{(\mathsf{A})})|\psi\rangle_{\mathsf{A}}$$
$$+\frac{1}{\sqrt{2}}|1\rangle_{\mathsf{M}}(\cos(\pi/8)\mathbb{1}+i\sin(\pi/8)\sigma_x^{(\mathsf{A})})|\psi\rangle_{\mathsf{A}}$$



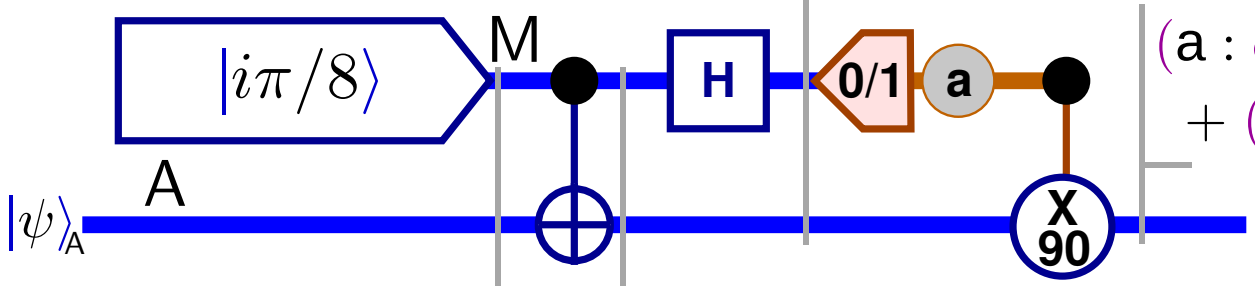
$$\cos(\pi/8)|0\rangle_{\mathsf{M}}|\psi\rangle_{\mathsf{A}} - i\sin(\pi/8)|1\rangle_{\mathsf{M}}\sigma_x^{(\mathsf{A})}|\psi\rangle_{\mathsf{A}}$$

$$\cos(\pi/8)|0\rangle_{\mathsf{M}}|\psi\rangle_{\mathsf{A}} - i\sin(\pi/8)|1\rangle_{\mathsf{M}}|\psi\rangle_{\mathsf{A}}$$

Rotations by State Preparation and Measurement

- $\mathbf{X}_{45^\circ}$ using $|i\pi/8\rangle = \cos(\pi/8)|0\rangle - i\sin(\pi/8)|1\rangle$.

$$\frac{1}{\sqrt{2}}\cos(\pi/8)(|0\rangle_{\mathsf{M}}+|1\rangle_{\mathsf{M}})|\psi\rangle_{\mathsf{A}}$$
$$-\frac{i}{\sqrt{2}}\sin(\pi/8)(|0\rangle_{\mathsf{M}}-|1\rangle_{\mathsf{M}})\sigma_x^{(\mathsf{A})}|\psi\rangle_{\mathsf{A}}$$
$$=\frac{1}{\sqrt{2}}|0\rangle_{\mathsf{M}}(\cos(\pi/8)\mathbb{1}-i\sin(\pi/8)\sigma_x^{(\mathsf{A})})|\psi\rangle_{\mathsf{A}}$$
$$+\frac{1}{\sqrt{2}}|1\rangle_{\mathsf{M}}(\cos(\pi/8)\mathbb{1}+i\sin(\pi/8)\sigma_x^{(\mathsf{A})})|\psi\rangle_{\mathsf{A}}$$



$$(\mathsf{a}:0)\frac{1}{\sqrt{2}}\mathbf{X}_{45^\circ}|\psi\rangle_{\mathsf{A}}$$
$$+\ (\mathsf{a}:1)\frac{1}{\sqrt{2}}\mathbf{X}_{45^\circ}|\psi\rangle_{\mathsf{A}}$$

$$\cos(\pi/8)|0\rangle_{\mathsf{M}}|\psi\rangle_{\mathsf{A}} - i\sin(\pi/8)|1\rangle_{\mathsf{M}}\sigma_x^{(\mathsf{A})}|\psi\rangle_{\mathsf{A}}$$

$$\cos(\pi/8)|0\rangle_{\mathsf{M}}|\psi\rangle_{\mathsf{A}} - i\sin(\pi/8)|1\rangle_{\mathsf{M}}|\psi\rangle_{\mathsf{A}}$$

Eigenvalues, Eigenvectors

- Let U be a unitary operator.
  - U is diagonalizable, $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$,

    with $|\psi_j\rangle$ orthonormal.

Eigenvalues, Eigenvectors

- Let U be a unitary operator.
  - U is diagonalizable, $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$, with $|\psi_j\rangle$ orthonormal.

$$\overset{U}{\begin{pmatrix} u_{11} & u_{21} & \dots & u_{N1} \\ u_{12} & u_{22} & \dots & u_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ u_{1N} & u_{2N} & \dots & u_{NN} \end{pmatrix}} =$$

$$\overset{V}{\begin{pmatrix} v_{11} & v_{12} & \dots & v_{1N} \\ v_{21} & v_{22} & \dots & v_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ v_{N1} & v_{N2} & \dots & v_{NN} \end{pmatrix}} \overset{D}{\begin{pmatrix} e^{-i\lambda_1} & 0 & \dots & 0 \\ 0 & e^{-i\lambda_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-i\lambda_N} \end{pmatrix}} \overset{V^\dagger}{\begin{pmatrix} \bar{v}_{11} & \bar{v}_{21} & \dots & \bar{v}_{N1} \\ \bar{v}_{12} & \bar{v}_{22} & \dots & \bar{v}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{v}_{1N} & \bar{v}_{2N} & \dots & \bar{v}_{NN} \end{pmatrix}}$$

Eigenvalues, Eigenvectors

- Let U be a unitary operator.
  - U is diagonalizable, $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$, with $|\psi_j\rangle$ orthonormal.

$$\overset{U}{\begin{pmatrix} u_{11} & u_{21} & \dots & u_{N1} \\ u_{12} & u_{22} & \dots & u_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ u_{1N} & u_{2N} & \dots & u_{NN} \end{pmatrix}} = \overset{V}{\begin{pmatrix} v_{11} & v_{12} & \dots & v_{1N} \\ v_{21} & v_{22} & \dots & v_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ v_{N1} & v_{N2} & \dots & v_{NN} \end{pmatrix}} \overset{D}{\begin{pmatrix} e^{-i\lambda_1} & 0 & \dots & 0 \\ 0 & e^{-i\lambda_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-i\lambda_N} \end{pmatrix}} \overset{V^\dagger}{\begin{pmatrix} \bar{v}_{11} & \bar{v}_{21} & \dots & \bar{v}_{N1} \\ \bar{v}_{12} & \bar{v}_{22} & \dots & \bar{v}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{v}_{1N} & \bar{v}_{2N} & \dots & \bar{v}_{NN} \end{pmatrix}}$$

  - There is a complete set of eigenvectors $|\psi_j\rangle = V|j\rangle$.

$$U|\psi_j\rangle = VDVV^\dagger|j\rangle = VD|i\rangle = e^{-i\lambda_j}V|i\rangle = e^{-i\lambda_j}|\psi_j\rangle$$

Eigenvalues, Eigenvectors

- Let U be a unitary operator.
  - U is diagonalizable, $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$, with $|\psi_j\rangle$ orthonormal.

$$\overset{U}{\begin{pmatrix} u_{11} & u_{21} & \dots & u_{N1} \\ u_{12} & u_{22} & \dots & u_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ u_{1N} & u_{2N} & \dots & u_{NN} \end{pmatrix}} =$$

$$\overset{V}{\begin{pmatrix} v_{11} & v_{12} & \dots & v_{1N} \\ v_{21} & v_{22} & \dots & v_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ v_{N1} & v_{N2} & \dots & v_{NN} \end{pmatrix}} \overset{D}{\begin{pmatrix} e^{-i\lambda_1} & 0 & \dots & 0 \\ 0 & e^{-i\lambda_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-i\lambda_N} \end{pmatrix}} \overset{V^\dagger}{\begin{pmatrix} \bar{v}_{11} & \bar{v}_{21} & \dots & \bar{v}_{N1} \\ \bar{v}_{12} & \bar{v}_{22} & \dots & \bar{v}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{v}_{1N} & \bar{v}_{2N} & \dots & \bar{v}_{NN} \end{pmatrix}}$$

  - There is a complete set of eigenvectors $|\psi_j\rangle = V|j\rangle$.

$$U|\psi_j\rangle = VDVV^\dagger|j\rangle = VD|i\rangle = e^{-i\lambda_j}V|i\rangle = e^{-i\lambda_j}|\psi_j\rangle$$

- Projective measurement of the eigenvalues of U?

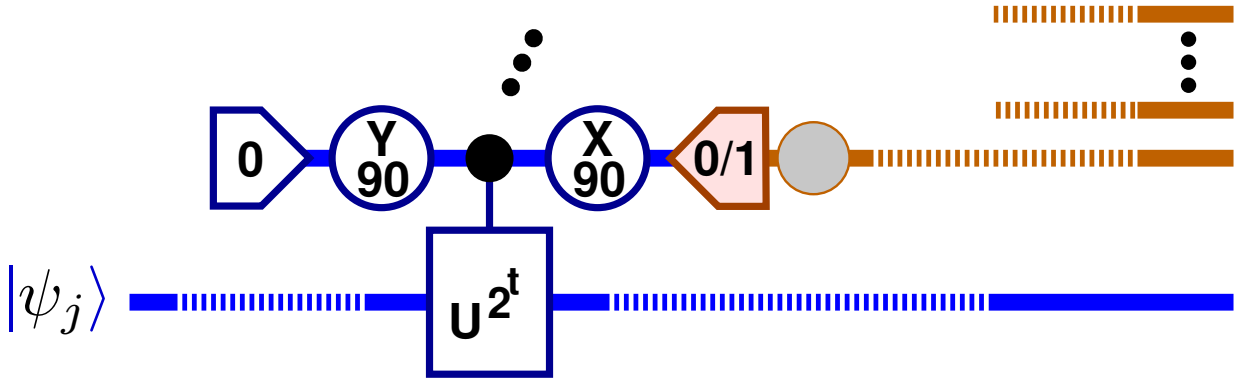
For distinct eigenvalues: $|\psi\rangle \to \left(\sum_j (\lambda_j \mapsto \mathsf{a}) |\psi_j\rangle\langle\psi_j| \right) |\psi\rangle$

Projective Eigenvalue Measurement

- Phase kickback for projective eigenvalue measurement. Assume:
  - U conditionally implementable.
  - $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$ with $|\psi_j\rangle$ orthonormal.
  - $\min(|\lambda_i - \lambda_j|) > \epsilon$.

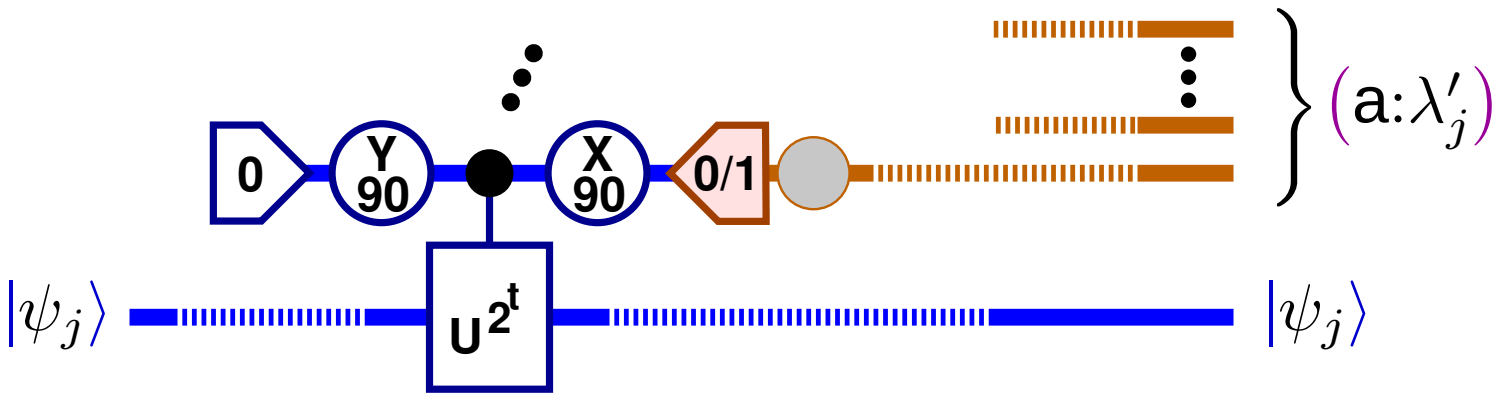
Projective Eigenvalue Measurement

- Phase kickback for projective eigenvalue measurement. Assume:
  - U conditionally implementable.
  - $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$ with $|\psi_j\rangle$ orthonormal.
  - $\min(|\lambda_i - \lambda_j|) > \epsilon$.



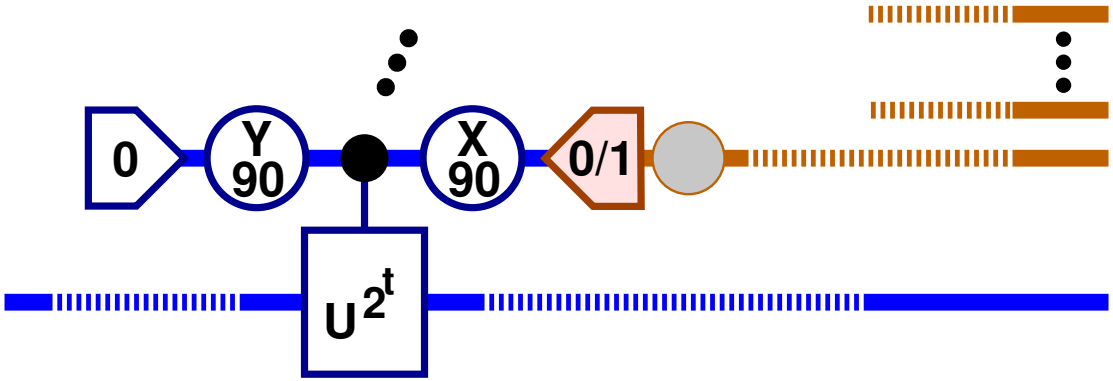
Projective Eigenvalue Measurement

- Phase kickback for projective eigenvalue measurement. Assume:
  - U conditionally implementable.
  - $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$ with $|\psi_j\rangle$ orthonormal.
  - $\min(|\lambda_i - \lambda_j|) > \epsilon$.



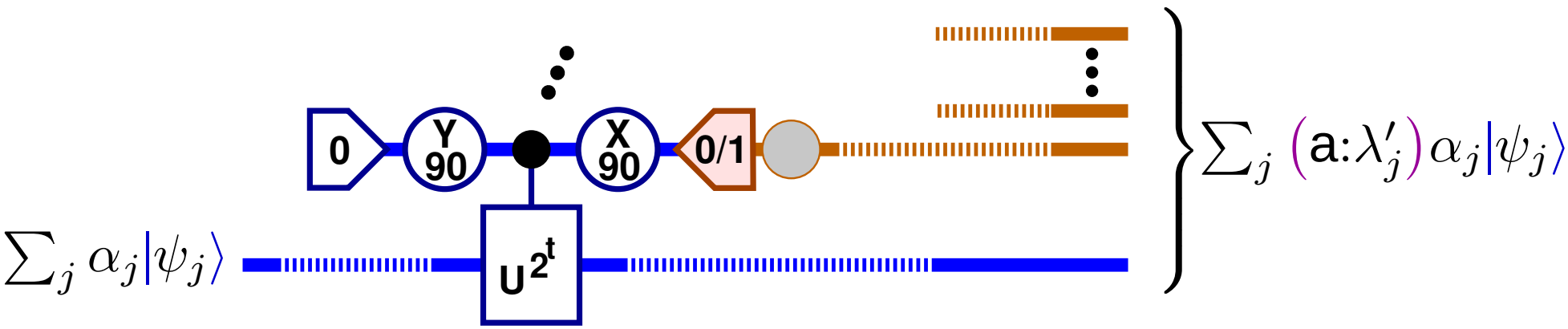
Projective Eigenvalue Measurement

- Phase kickback for projective eigenvalue measurement. Assume:
  - U conditionally implementable.
  - $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$ with $|\psi_j\rangle$ orthonormal.
  - $\min(|\lambda_i - \lambda_j|) > \epsilon$.



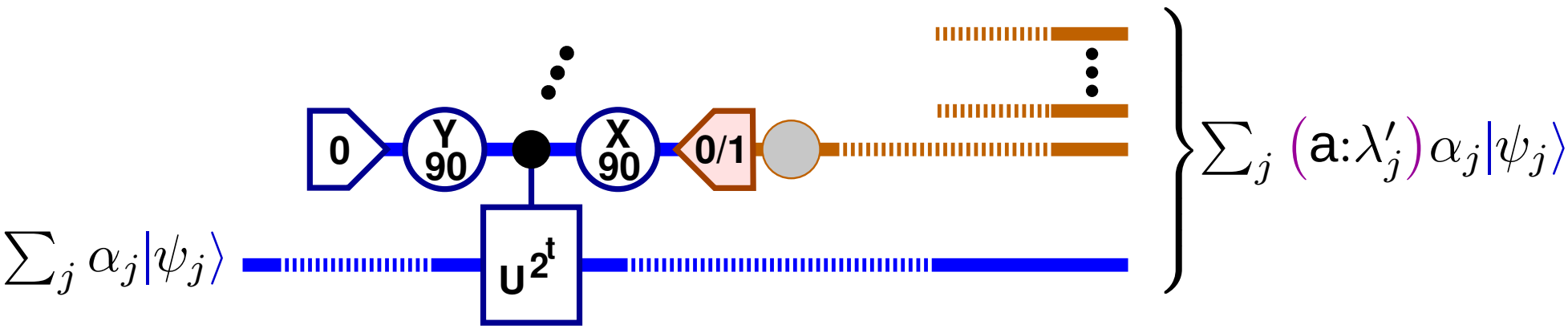
Projective Eigenvalue Measurement

- Phase kickback for projective eigenvalue measurement. Assume:
  - U conditionally implementable.
  - $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$ with $|\psi_j\rangle$ orthonormal.
  - $\min(|\lambda_i - \lambda_j|) > \epsilon$.



Projective Eigenvalue Measurement

- Phase kickback for projective eigenvalue measurement. Assume:
  - U conditionally implementable.
  - $U = \sum_j e^{-i\lambda_j} |\psi_j\rangle\langle\psi_j|$ with $|\psi_j\rangle$ orthonormal.
  - $\min(|\lambda_i - \lambda_j|) > \epsilon$.



  - Requires $O(\log(1/\epsilon) \log\log(1/\epsilon))$ measurements.
  - Can implement $\simeq \sum_j (\lambda'_j \mapsto \text{a}) |\psi_j\rangle\langle\psi_j|$ with $\lambda'_j = \lambda_j \pm \epsilon/2$.

Contents

Title: IQI 04, Seminar 6 0
Distinguishable One-Qubit States . . . . . top. . . 1
Bra-Ket Algebra I . . . . . top. . . 2
Bra-Ket Algebra II 3
Ket-Bra Expressions I . . . . . top. . . 4
Ket-Bra Expressions II . . . . . top. . . 5
Ket-Bra Expressions III . . . . . top. . . 6
Ket-Bra Expressions IV . . . . . top. . . 7
Overlap . . . . . top. . . 8
Measuring One Qubit Among Many I . . . . . top. . . 9
Measuring One Qubit Among Many II . . . . . top. . . 10
Projective Measurement . . . . . top. . . 11
Rotations by State Preparation and Measurement I . . . top. . . 12
Rotations by State Preparation and Measurement II . . top. . . 13
Eigenvalues, Eigenvectors . . . . . top. . . 14
Projective Eigenvalue Measurement . . . . . top. . . 15
References 17